Structures, Learning and Ergosystems: Chapters 1-4, 6.

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Contents

Abstract

We introduce a concept of *an ergosystem* which functions by building its "internal structure" out of the "raw structures" in the incoming flows of signals.

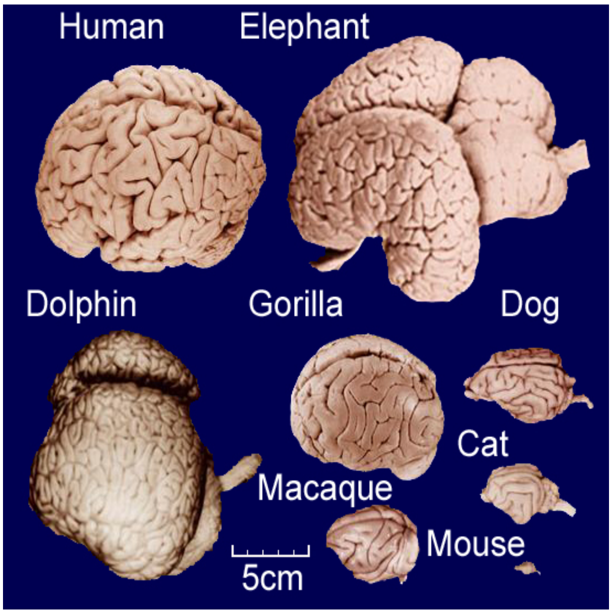
1 Ergobrain and Evolution.

We shall argue in this section that the essential mental processes in humans and higher animals can be represented by an abstract universal scheme and can be studied on the basis of few (relatively) simple principles.

1.1 Ergo, Ego and Goal Free Learning.

The brain of a human or an animal, for ever confined to the scull, has no direct access to the outside world. The "outside world" for the brain is nothing else but a muddy flow of electric/chemical signals it receives from receptor cells and the endocrine system(s). The brain responds with similar signals directed mostly to the muscles of the body. (Some signals go to the brain itself and some to the endocrine glands in the body.) Here is a vivid description by Charles Sherrington [80]:

"The eye sends, as we saw, into the cell-and-fibre forest of the brain throughout the waking day continual rhythmic streams of tiny, individually evanescent, electrical potentials. This throbbing streaming crowd of electrified shifting points in the spongework of the brain bears no obvious semblance in space pattern, and even in temporal relation resembles but a little remotely the tiny two-dimensional upside-down picture of the outside world which the eyeball paints on the beginnings of its nerve fibers to the brain. But that little picture sets up an electrical storm. And that electrical storm so set up is one which effects a whole population of brain cells. Electrical charges having in themselves



not the faintest elements of the visual - having, for instance, nothing of "distance," "right-side-upness," nor "vertical," nor "horizontal," nor "color," nor "brightness," nor "shadow," nor "roundness," nor "squareness," nor "contour," nor "transparency," nor "opacity," nor "near," nor "far," nor visual anything - yet conjure up all these. A shower of little electrical leaks conjures up for me, when I look, the landscape; the castle on the height, or, when I look at him approaching, my friend's face, and how distant he is from me they tell me. Taking their word for it, I go forward and my other senses confirm that he is there".

Ergo. In sum, the transformation of signals by the brain can be depicted by an arrow

$$\textit{incoming signals} \rightsquigarrow \textit{outcoming signals},$$

which, metaphorically, corresponds to "ergo" in the Cartesian "Cogito, ergo sum".

We call the (hypothetical, formal) structure implementing this transformation an *ergobrain* or an *ergosystem.* We think of this as a dynamic entity continuously being built by the brain from the moment of birth of a human or an animal by the process of *(goal free) structure learning.*

Very roughly, an ergosystem is something like a network of "ergo-ideas" which grows in the course of learning by incorporating and "metabolizing" new "ideas" from the incoming flows of signals.

But from ergobrain's own perspective, the "real world" emerges via the reversed arrow symbolizing the transformation from the signals the ergobrain generates to their "echos" – the responses to these signals. The ergobrain learns by analyzing/structuralizing the "information" carried by these echos – "patches of ripples" on the external flows of signals resulting from ergobrain's "actions", where some "echos" originate within the ergobrain itself.

The simplest instance of learning performed by the ergobrain is conditioning: the process of acquisition of conditional reflexes. We are interested, however, in more "complicated" mental processes, e.g. acquisition of native languages by children. Such processes performed by the ergobrain, be they simple or complicated, are unconscious but they leave some perceptible traces in our conscious mind (we shall explain this later on).

We take an avenue of mathematical, albeit informal and/or unrigorous, reasoning and try to argue "on brain's terms" with a minimal reference to the "meaning" which our "mind" assigns to these signals. (In practice, this is as difficult as not assigning, albeit unconsciously, any purpose to evolution. In Haldane's words: "Teleology is like a mistress to a biologist: he cannot live without her but he's unwilling to be seen with her in public".)

We do not hypothesize on how this arrow is implemented by the synaptic network of the brain, although we try to keep track of restrictions on possible structures of "$\leadsto$" which are due to limitations of network architectures. And we constantly keep in mind possible algorithmic realizations of ergosystems.

Neuro-brain. This signifies a (semi-mathematical) representation (model) of the physiology of the brain, including "streaming crowd of electrified shifting points in the spongework of the brain'" along with a (the ???) slower biochemistry of middle-term and long-term memory.

An ergobrain can be thought of as a (kind of) *dynamical reduction* (quotient) of a neuro-brain, while an ergosystem is something similar to an ergobrain but which is not, a priori, related to a neuro-brain.

Ego-mind. This refers to (manifestations of) mental processes (more or less) *directly* serving and/or reflecting the *survival/reproduction* needs of an organism. Ego-mind is *pragmatic* and it is *goal oriented*, where the goals are set up by the *evolutionary genetic (pre)history* of an (animal or human) organism and they are modulated by interactions of an organism (or of its mind) with the *outside world.*

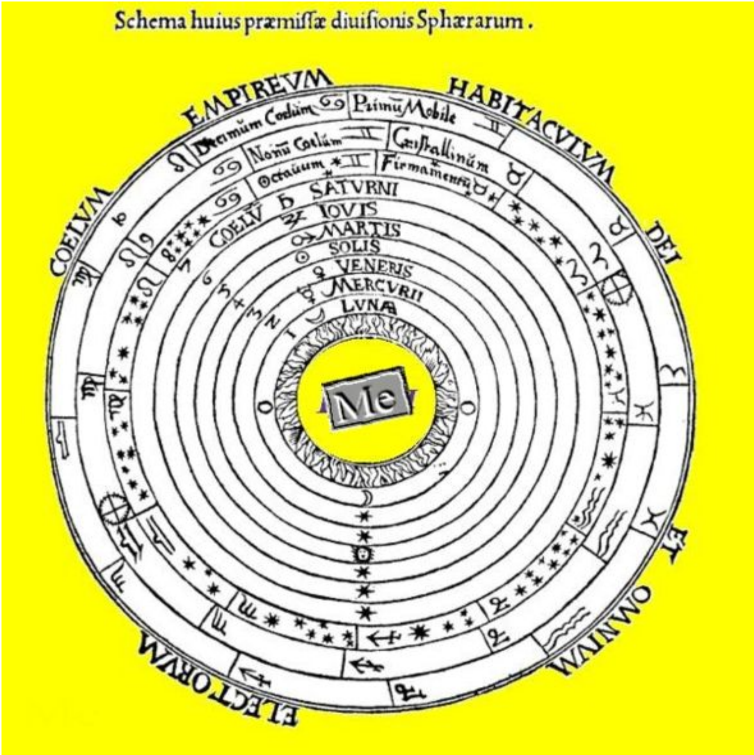
Most (but not all) of what we consciously perceive is produced by the ego-mind. (The Freudian type subconscious and most of what we see in dreams are byproducts of the ego-mind.) In these terms, the ergobrain includes the totality of mental processes which are not covered by the ego-mind as well as some "ego-parts", such that the mechanisms of conditional and unconditional reflexes which were, apparently, *directly* targeted by the evolution selection process.

The egomind and its *ego-reasoning* (common sense) are adapted to the every day life, they stay on guard of our survival and passing on our (and of close kin – ego can be altruistic) genes. "Myself" is the central reference point in the visible part of our mind as much as the sun is central for accounting for the observed planetary orbits. But as the centrality of the sun is secondary to such concepts as "differential equation" and "Hamiltonian system", the centrality of ego is secondary to the (unknown to us) "laws" running general ergosystems.

Besides, our ergomind was not designed by evolution for a structural modeling of the world. The self glorifying ego-vocabulary:

"I think", "I am", "important" "intelligent", "understanding", "conscious", "intuitive", "insightful", "creative", "motivated", "purposeful", "rational", "adaptive", "efficient", "resourceful", "rewarding", "useful", etc.

is not adequate for *structural modeling* of most essential mental processes, such as *learning*; we shall do our best to keep our distance from these "concepts"



except for a metaphorical usage of the words.

Goal Free Learning. The ergobrain, as we shall argue, plays the key role in the learning processes, where our notion of learning is far from the concept of "adaptation" and it does not essentially depend on any external "reward/punishment" reinforcement. Our ergobrain models are "self propelled learners" which need no additional external or internal reinforcement stimuli, similar to some current robotic models, e.g. as in [72]. Ergobrains and ergosystems are structures (evolutionary or artificially) designed for

recognizing, selecting, analyzing, interacting with and building structures;

(ergo)learning is a spontaneous structure building and rebuilding process.

We do not necessarily exclude an influence of "learning reinforcement stimuli", both, of "nature", i.e. genetically inherited by an organism, and of "nurture", i.e. modulated by the environment. These may change the learning behavior, similarly to a change of motion of a free particle system under a constrain or an external force.

For example, people possess an ability do recognize human faces (which does not automatically extends, however, to faces of people of a remote ethnicity). This may be inborn but it may be due to the fact that mother's face is the most elaborate and structurally interesting visual pattern encountered by a baby and analyzed in depth by baby' ergobrain.

(Were a baby mainly exposed to images of the Mandelbrot set, he/she could grow up forever attached to the fractal symmetries, rather than to the bilateral ones, as much as ducklings take the first moving object they see upon hatching for their mother.)

Arguably, the *essential* mechanism of learning is goal free and independent of any reinforcement, as much as the mechanical motion *essentially* depends on *the inertia* and but not on the constrains and external forces. (This agrees with the ideas expressed by child psychologists about fifty years ago, [8] [89].)

Our aim is to find/construct such models of ergobrains and learnings that can be (potentially) described as *dynamical reductions* of neuro-brains. Also we search for model(s) of ergobrains which are sufficiently *universal.* This is opposite to what is expected (of models) of ego-minds and neuro-brains, since ego-minds need to serve *diverse* specific environmentally imposed purposes, while the (up to a large extent, random) architectures of neuro-brains are "frozen accidents" in terms of Frances Crick. But, as we shall argue, these diversity and accidentality necessitate the universality of ergobrains. (Our models are very much in spirit of what is currently called "intrinsically motivated reinforcement learning", see [55] [10] and references therein.)

About Structures. We shall be talking much about "structure" without ever giving a definition. The only way we can explain what it means is by providing examples; these will come as we proceed. Below are two instances of structures.

The grammar of a natural language is an instance of a structure. It is associated to our ergobrain and a native speaker is unaware of the fact that he/she speaks grammatically. On the other hand *pragmatics* of the language is, to a large extent, the product and the manifestation of the egomind. The correspondence *ergo/ego* $\leftrightarrow$ *grammar/pragmatics* is not quite faithful, since the grammar is essentially static and it is smaller in volume than the pragmatics, while the ergobrain is a dynamic structure and most of the brain mental dynamics is occupied with ergo (e.g. generating sentences) rather than with ego.

The living cell may be seen as a chemical system where small molecules undergo a variety of transformations.

The (mostly chemical) interaction with the environment is the "ego-activity" of the cell while selection of beneficial mutations of DNA corresponds to the neurophysiology of learning.

Here a cytologist/bacteriologist (chemist?) plays the role of a psychologist while the place of a neurophysiologist is taken by a molecular geneticist who tries to understand how variations of the sequential composition of DNA and/or of the production of mRNA influence the chemical behavior of a cell, similarly to how a (dis)activation of a specific region of the brain and/or of an individual neuron affect the psychological state.

What is missing from this cytologist + geneticist picture is biochemistry of proteins that run the bulk of the cellular processes.

Back to the brain, we conjecture, that there is a vast and intricately structured "something" that makes the interface between neurophysiology and psychology. We look for this "ergo-something" embedded into a mathematical frame where this "embedding" should be no more mathematically perfect, than, for example, a rigorous positioning of biochemistry of proteins within the main body of chemistry.

However, there is an essential difference between the cell and the brain: all molecules, be they DNA, proteins or mundane H_2O, live in the same world: the cell. But the "ideas" of the ergobrain are not of the mind nor do they live in the "physiological space" – the world of neurons, synapses, electro-chemical currents, neurotransmitters, etc. – the two worlds do not even overlap. Ergobrain and its "ideas" are (quasi)mathematical entities, which inhabit (something like) a quotient (reduction) of the neurobrain, not the neurobrain itself. (A rough counterpart to the *structure* of the ergobrain is the graph theoretic *combinatorics* of metabolic pathways and of protein interaction networks in cells.)

It is unimaginable that a viable representation of the brain/mind interface is possible without being incorporated into a broader mathematical framework. Yet it would be equally naive to expect such a representation with no new significant input from experimental psychology and neurophysiology.

One may be encouraged (discouraged?) in a search for such a framework by the history of discovery of genes – units of inheritance, that were originally conceived by Gregor Mendel as *abstract mathematical entities.*

Mendel, who had studied physics at the University of Vienna under Doppler, famous for the *Doppler effect*, and Ettigshausen, known for his *Combinationslehre*, had a fine feeling for mathematical structures; however, he had not arrived at the idea of genes by a purely abstract reasoning but rather on the basis of thousands experiments with interbreeding pea plants which he exposed in the 1866 paper "Versuche über Plflanzenhybriden".

Genes "materialized" in 1953 with the identification of the DNA structure by Crick and Watson who also conjectured that when a cell divided each of the daughter cells inherited one of the two mother's DNA strands and synthesized the second strand anew. This was confirmed in 1957 by Meselson and Stahl:

Bacterial (E. coli) cells were grown for several generations in a nutrient medium with heavy nitrogen ^{15}N instead of ^{14}N and then transferred to the normal ^{14}N-medium. It was shown with centrifugations, that, after two divisions, half of the cells had DNA of normal light weight and half had the weight intermediate between normal and that in the first "heavy" ^{15}N -generation. See *http : //www.cap – lmu.de/fgz/portals/biotech/timetable.php* for the chronology of the main steps of genetics.

The diversity of functions performed by the proteins necessitates the universality and "overall simplicity" of this "proto-ergo": evolution is not that clever to design several different production lines of "smart molecules".

(Well... what about ribozymes? And "overall simplicity" is far from simple: it took, apparently, a half billion years for Nature to make the ribosome out of RNA and proteins; and once was enough.)

"In principle", this "proto-ergo" could be reconstructed from a *detailed* knowledge of the chemistry and (bio)physics but no such knowledge is anywhere in sight; nor would it help unless you have an idea of the possible conformations and functions of large molecules.

But one still has a limited understanding of water on the nanoscale, one has no realistic physical model of protein folding, one has no quantum-chemical explanation for the catalytic efficiency of enzymes, etc. – the situation with the brain is no better.

However, even what is known to-day, even a roughest outline of the available (huge) body of knowledge, may direct you toward amusing (quasi)mathematical theories.

Acknowledgment. My "understanding" of anything beyond a few branches

of mathematics is no match to "non-understanding" by the true students of the respective fields. I attempt, however, to present, what I have heard from biologists/biophysicists and picked up from non-technical literature, in a formate which is, hopefully, amenable to further mathematical formalization.

Our condensed rendition of biologists' ideas (such as the above $^{14}\mathrm{N} \mapsto {}^{15}\mathrm{N}$) can make them look deceptively simple, if not naive. The catch is, as Maxim Frank-Kamenetskii once put it to me, that at most 1% of *brilliant* ideas turn out to be *right* in biology (it is, I guess, close to 90% in mathematics) while only few ideas can be tested in a realistic stretch of time: the amount of skilled work going into a serious experiment can hardly even be imagined by a mathematician like myself not to speak of enormity of the background knowledge needed to design such an experiment.

(Unsurprisingly, some ideas had to wait before they were taken in earnest. For example, Alexander Fleming stopped working on therapeutic use of extracts from Penicillium notatum in favor of his other projects; the development of penicillin was initiated ten years later by Howard Walter Florey and Ernst Chain [66].)

Artificial Intelligence, Connectionism and Ergo. There are two current approaches to modelling mind and brain.

1. *The (classical) A.I.* ($http://plato.stanford.edu/entries/logic\text{-}ai/$ and [42]) is concerned with a representation of the *behavior* of the *ego*mind. This, in the metaphoric cell language, is designing industrial plants for productions of the small molecules which the cell produces.

2. (Broadly understood) *connectionism* is an engineer's construction of models of the *neurobrain.*

($http://plato.stanford.edu/entries/connectionism/$ and [21], [53], [38]). This is similar to making artificial entities with cell-like architectures and then teaching them cell-like chemical behavior by adjusting their DNA and mRNA.

The logic of our "ergo-approach" can be already seen in Poincare's sketch of a (nearly mathematical) proof of *non-possibility* of reconstruction of the geometric structure of the space, e.g. the invariance of the geometry under rigid motions (isometric transformations), *without* an arsenal of motions available to a learning system [78].

Poincare suggested that a movement of the retinal-image triggers a "neuronic recall" of the corresponding eye movement and conjectured the existence of "something in the brain" (*mirror neurons*?) which similarly reacts to an observer's eye and to an observed object movements.

(Poincare had arrived at this conclusion long before it was discovered that the retinal receptors responded primarily to *fluctuations* of illumination and that the retinal-image movements were essential to vision: when the retina receives a stabilized image, the structure of the perceived target fades out in a couple of seconds. Also, Poincare could not know that dreams in the REM phase of sleep were accompanied by eye movements, while the primary visual cortex remained inactive.)

Our intended ergosystem models will be rather *universal* and/or *canonical*, depending on (relatively) small numbers of arbitrary "parameters". We do not need anything like a list of "axioms of folk physics" as some A.I. systems need for programming their robots [49]; nor do we have to imitate the fine details of the architectures of specific models of the neurobrain. Metaphorically, a "newly born" ergosystem is like a stem cell that specializes in the corse of chemical interaction with the environment (which is mainly determined by other cells.)

The idea of modeling human mind by a formal system is presented by Turing in his 1950 paper "Computing machinery and intelligence" [83]; Turing's argument in favour of the existence of such a model remains unchallenged.

Yet, there are possible several renditions of Turing's idea depending on how you understand the words "formal system" and "intelligence". The two sources nearest to what we attempt to achieve are Alexandre Borovik's book on perception of mathematics by mathematically inclined children [11] and design of self motivated learning robots pursued by Pierre-Yves Oudeyer's team [4], [73], [74]; also see "intrinsic motivation" [6] http://www-all.cs.umass.edu/ barto/

My Apology to non-Mathematicians. It seem to me that the basic mental processes can be *meaningfully* described, if at all, *only* in a broader *mathematical* context and this mathematics does not exist at the present day.

The role of this, still non-existing, "abstract mathematics" might be compared to that of *Hilbert, spaces, tensor products, self-adjoint operators...* – in packaging concepts of quantum mechanics. Conceivably, one can convey these ideas in a less intimidating language but I am aware only of a single successful instance of this : QED by Richard Feynman.

Yet, nothing can make it easy. If it feels light – you are building your mental muscles with empty plastic dumbbells and you end up with polystyrene foam in your head rather than an imprint of the true structure.

The fragments of mathematical (and some non-mathematical) structures in the first chapters of our book are intended as pointers to models of ergosystems which will come later on. The reader browsing through the ranges of mathematics would better skip whatever he/she finds incomprehensible, and, if interested, would return to it later on. It takes 5-10 mental regurgitations over a several months time interval to assimilate an entirely new non-trivial idea, be it a mathematician's or non-mathematician's ergo-brain, while keeping your smart ego-mind at bay.

Ergo-Chess. An example of "ergo-behavior" which we intend to design is as follows. Suppose, an ergosystem is shown several (possibly many) chess games, and/or fragments of games, where some are true games by chess players and the other are "junk", random moves compatible with the rules of the chess. (Some logicians take such "random axiomatic junk" for the model of mathematics.) Our system will *recognize* the real games – it will regard these as "interesting", will *single them out*, will *learn* (remember) them and try to *imitate* the moves of the players. Eventually, the system will start *teaching itself* to play better and better *without* any additional stimuli.

And the same system will behave similarly if it is shown any other *structurally interesting* game, e.g. checkers or the game of Go, tennis or instances of *dialogs* in a natural language.

Conversing in a natural language is, metaphorically [81], just another kind of game where much (all?) of the "*real* meaning of words" is derived from the structural patterns of the language similarly to the "*real* meaning of moves" in a chess game, where none of the two "realities" (directly) connect to the "real world". (Human languages, unlike "animal languages" and games like chess, are saturated by "self-referential loops", which, paradoxically, facilitate identification of the overall structure of a language.)

A characteristic feature of a "pure-ergo" system (unlike the biological *ergo+ ego*) is that it *does not try to win* a game, "winning" is an ego-notion: the system will adjust to a (possibly weaker) player and will try to make the game as interesting (to itself) as it can. (This may be not very "interesting" to the second player, as, e.g. in the cat and mouse game.)

This example leads us to another concept, that of an *interesting structure* and/or of an *ergo-idea*: this is something an ergobrain recognizes as such and then starts learning this structure (idea) by incorporating it in its own structure. (A learning by a biological organism, also involves an acquisition of ego-ideas, but these carry little structure of their own and have no significant effect on the structural modifications of the ergobrain.)

We (usually) identify: *structure = interesting structure.* To close the circle, we declare that

the goal free structure learning is a structurally interesting process.

Eventually, the very circularity of these "definitions" (similar to Peano's axiomatic "definition" of numbers) will bring us toward constructions of specific models of ergosystems, relying on and starting from our mathematical/scientific (ergo)perception of

interesting, amusing, surprising, amazing and beautiful structures,

where the corresponding *ergo-moods*, such as being "curious", "interested", "perplexed" "bored", etc. appear as *the signatures* of such perceptions.

What Skeptics Say. Frances Crick states in the epilog to his book What Mad Pursuit: "Intellectual snobbery makes them (mathematicians?) feel they can produce results that are mathematically both deep and powerful and also apply to the brain".

Francis Crick, Gregor Mendel, Charles Darwin spent their lives in the "mad pursuit" of *beautiful structures* in biology – they were mathematicians at the bottoms of their hearts but the abstract mathematics of their days was never of any help to them. Yet, one should never be discouraged by *anybody's* "never".

1.2 Peacock Tail, Maxwell Equation and $E = mc^2$.

Orgel's second law (an excuse for a missing explanation) says: "Evolution is cleverer than you are". Let us reformulate this by saying: "We are even stupider than Evolution is".

Evolution is "big" it may select out of billions of organisms for billions of generations, but it is structurally shallow: "randomly modify whatever you have got and weed out whoever does not pass the survival/reproduction test". Evolution can not make miracles on its own: it proceeds in relatively simple (not all of them "small") steps. The outcome looks miraculous to our minds only because we can not guess, what kind of *mathematical* structures govern/constrain these steps, how mathematics is implemented by physics and chemistry and how these steps add up (within yet another mathematical/physical/chemical structure) to "miracles". Only exceptionally, evolution creates totally new complicated structures, such as the genetic code and ribosome, and only if it concentrates on the task very seriously.

In the following sections we bring up instances of human and animal behavior which are, on one hand, miraculously complicated, on the other hand they have little, if any, pragmatic (survival/reproduction) value. From this we conclude that since the corresponding features of ergobrains were *not the primarily targets* specifically selected for by the evolution, they are due to *internal constrains on possible architectures* of unknown to us functional "mental structures". The following (non-mental) example serves only as an illustration.

Peacock Tail. One can say that the peacock tail has evolved, because the peahen selects beautiful males for mating. But almost everything can be "explained" this way! (I am uncertain if there is a convincing study which indicates that the brilliance of a male's plumage meaningfully correlates with his mating success; yet, we stick to the idea that this beauty is for another bird's eye.) To be taken seriously, an explanation needs to include plausible bounds on the following quantities:

(1) a bound on the "information content" of the color display on the tail and then on the number of genes (out of total $\approx 30\ 000$?) which are dedicated to encoding this information;

(2) a similar bound on "something" in the bird's visual system that can "read off" and "understand" what is "written" on the peacock tail.

As for (1), one at least knows what the *starting point* of a *structural* explanation might be: brilliant colors of the peacock plumage are due to the optical interference (Bragg reflection) based on (nearly) *periodic nanostructures* in the fiber-like components of the feathers. Such interference-based structural color produces the peacock's iridescent hues (which shimmer and change with viewing angle), since interference effects depend upon the angle of light, unlike chemical pigments (Wikipedia).

All this has nothing to do with sex and little with selection. The sexual selection, so to speak, took a ride on several pre-existing structures: the geometry of space (which allows periodic structures), on wave optics (Maxwell equations), and on the (selected beforehand) sensibility of the bird's eye to the spectrum of the visible light. Evolution, if selecting pixel by pixel, would not make a single feather of the bird.

Nature's pawns of the mutation/selection game are billions upon billions but their number is dwarfed by the the ratio of the number of nonfunctional programs of animal behavior to the number of the consistent/successful ones. Besides, Nature does not gently guide her subjects toward any goal but just weeds out the descendants of those who poorly scored at the selection test, or who, such as Neanderthals for instance, had their files moved to the extinction folder by an oversight of Nature.

All this has to be reconciled with the emergence of complicated and efficient behavioral patterns (such as the well studied rat grooming sequence, for instance [18]). Some geometric/combinatorial properties of the totality of the successful programs must compensate for this discrepancy.

Thus, turning to (2), one may conjecture that (certain patterns in) the bird's visual processing system(s) is organized according to a relatively small class of (nearly) universal structures (comparable to periodicity in their complexity). This would limit the pool of selection (of such patterns) and make evolution plausible.

Alfred Russel Wallace, Darwin's rival as the author of the natural selection theory, did not believe that the sexual selection could *fully* account for the higher mental functions in humans [86]. He maintained that there must be "something transcendental" responsible for the emergence of higher cognition. We suggest that "Wallace's something" is a class of *evolutionary/biologically accessible "mathematical ergo-structures"* (with greater possibilities than, say, the grooming program in rats) and we shall bring an evidence for the existence of such structures in this paper.

But could one argue in favor of "something like Maxwell equation" on the basis of the sexual selection of the peacock tails? Preposterous as it seems, there is a historic precedent when such reasoning turned out to be valid.

Benot de Maillet (1656 – 1738) estimated the age of Earth at ≈ 2 billion years, on the basis of his (reasonably incorrect) ideas of development of the earth crust and the rates of sedimentation [5]. (The current estimate of ≈ 4.5 billion years has been determined by radiometric dating of meteorite material and the oldest terrestrial and lunar samples.)

Charles Darwin and his contemporary, a geologist Charles Lyell, each made guesses as to how long ago certain fossils formed. Darwin dated the end of the Cretaceous period at 300 million years ago (the currently accepted time is 65 million). Lyell placed the start of the Ordovician period at 240 million years ago; (the current estimate is ≈ 500 million).

On the other hand, the great physicist William Thomson, first estimated the age of Earth at between 20 million and 400 million years and eventually (1897) settled on 20-40 million years [13]. This, similarly to the earlier 75 000 years estimate by by Georges Buffon (1707 - 1788), was based on an evaluation of the time needed for Earth to cool from the molten state. Amusingly, Thomson's calculation was in a good agreement with those by Helmholtz and Newcomb of the time for the Sun to be heated and sustained by the gravitational contraction.

(Buffon – the great naturalist of the 18th century who initiated a critical scientific approach to the idea of evolution and (justifiably?) rejected this idea on the basis of the data available to him, is known to mathematicians as a discoverer of the stochastic geometry, including the Crofton formula expressing the length of a planar curve C by the integral of the number of intersections of C with straight lines. According to some story he tried to experimentally measure π by throwing French baguettes over his shoulder onto a tiled floor and counting the number of times the loaves fell across the lines between the tiles.)

In order to accept the estimates by Maillet, Darwin and Lyell, the physicists had to assume that there was an unknown to them source of energy keeping Sun/Earth warm. Radioactivity was discovered by Becquerel in 1896 with the first clue to its energy suggested by Einstein's $E = mc^2$ of 1905. Then in 1919, Henry Norris Russell, an American theoretical astronomer, summarized the evidence for the nuclear nature of the star energies.

In a way, Maillet, Darwin and Lyell have predicted that something like $E = mc^2$ must be in the stars!

Darwin himself did not seem to care about mc^2. But with almost five extra zeros after 75 000 at his disposal, he argued, that since the time variable enteres the *exponential* function – the number of potential descendants of an organism, the inevitable cut-off of this function, called by Darwin *natural selection*, i.e. the cut-off executed by Death, and, in a presence of the sex reproduction, by Extinction (called "sexual selection" for the sake of civility) provide a sufficient justification for the observable rate of evolution. (Darwin's argument follows in steps of Malthus' minimal wage survival theses in economy, where the idea, can be traced to Condorset.)

No additional intrinsic mechanism is needed – time-additive accumulation of inheritable small beneficial phenotypic variations should suffice according to Darwin.

This, with an actual evaluation of the rate of evolution shoveled under the carpet, gets you around the problem which confounded Jean-Baptiste Lamarck (1744-1829) – the proponent of the modern concept of evolution, who could not justify his ideas without plunging into alchemical speculations on the propensity of physical matter for self organization. Lamarckian "self organization alchemy" was met with scorn by his former student Jean Cuvier, e.g. in his "Elegy of Lamarck". Cuvier (1769-1832), the founder of paleontology, opposed the whole idea of "evolution by small variations" on the basis of his extensive experience with fossil material.

Prior to the Darwin's fundamental "On the Origin of Species" of 1859, the concept of "evolution by selection" was articulated by Pierre de Maupertuis (1698-1759), the author of the *principle of stationary action* in the classical mechanics, as follows:

"Chance one might say, turned out a vast number of individuals; a small proportion of these were organized in such a manner that the animals organs could satisfy their needs. A much greater number showed neither adaptation nor order. These last have all perished – thus the species which we see today are but a small part of all those that a blind destiny has produced."

Similar ideas were expressed in 1790s by Erasmus Darwin – Charles' grandfather, by Scottish-American physician William Charles Wells in 1818, by Scottish fruit farmer Patrick Matthew in 1831, and by English zoologist Edward Blyth in 1837.

Well, well,... but if you try hard, you can find such thoughts in the writings by Abu Uthman al-Jahiz (781-869):

"Animals engage in a struggle for existence; for resources, to avoid being eaten and to breed. Environmental factors influence organisms to develop new

characteristics to ensure survival, thus transforming into new species. Animals that survive to breed can pass on their successful characteristics to offspring." (Wikipedia's article refers to the source of this citation as unreliable.)

1.3 Eyes, Wings, Energy, Information and Icosahedra.

The peacock tail derivation of the Maxwell equation may appear far-fetched but, for example, the *spheroidal* structure of all vertebrates eyes (including humans) and some cephalopods (e.g. squids), which are constructed like simple cameras, directly points to the laws of the geometric optics.

Human eyeball is almost perfectly spherical which allows three rotational degrees of freedom (where, apparently, only two are effectively used.) Human focus by changing the shape of their lenses, squids move the lens backwards and forwards.

(Squids eyes are not perfectly spherical. I could not find out on the web whether they can rotate being surrounded by softer tissues or it is the lenses that move around.)

Our last common ancestor with squids, probably, had a simple photoreceptive spot, thus, the human/squid eyes structural similarity (universality) is essentially due to the constrains imposed by the geometry and optics of our (flat Euclidean) space.

(The *compound eye* structure in arthropods, e.g., insects, apparently, reflects the constrains of their evolutionary history. The compound eye has poorer resolution, but it allows an assimilation of changes faster than camera eyes do and its anatomy/phisiology matches the "simple" signal processing by an insect's brain.)

The anatomy of wings provides another example of universal structural constrains, imposed by the aerodynamics in this instance. Wings were modified from limbs of the last shared ancestor of birds and bats (as evidenced by their bone structure) but the overall architectural and functional similarity (universality) of birds/bats wings is due to the requirements of aerodynamics.

Insects – the only invertebrates who fly – have different wing architectures and flying patterns, such as hovering, which is possible for the small size (thus, small Reynolds number). The wingbeat of small Midges is ≈1000 beats/sec, while the the slowest insect wingbeat – of Swallowtail butterfly – is ≈5 beats/sec. (Our understanding of the aerodynamics of flexible, flapping wings and how insects fly remains incomplete.)

Hummingbirds, whose weights range 2 - 9g, have their fly modes, e.g. hovering, similar to, but distinct from, that of insects. The small ones fly with about 20-40 beats per second which may go up to 200 during courtship displays.

One may only speculate whether the overall structures of camera eyes and functional wings could have been derived solely from general principles of physics and geometry.

But in 1956, Crick, Watson, Caspar and Klug [28] [15] [29] [16] [58] had *predicted* possible *icosahedral symmetry of viruses* on by an essentially *mathematical* reasoning partly based on (rough at that time) analysis of x-rays diffraction on crystals of viruses.

(Icosahedron, one of the five *Platonic bodies*, is a convex polyhedron with 20 triangular faces, 30 edges and 12 vertices, where each vertex has 5 faces adjacent to it. Its symmetry group consists of 120 elements: 60 rotations around the center and $60 = 30 + 30$ mirror reflections with respect to the 30 planes normally bisecting the edges and 30 bisecting the dihedral angles at the edges. This is, in a sense, the maximal rotational symmetry available to a polyhedron in the 3-space.)

Why does the *random* mutation/selection evolutionary mechanism generates *improbable* symmetries? There are two reasons for this, where the first one has little to do with evolution.

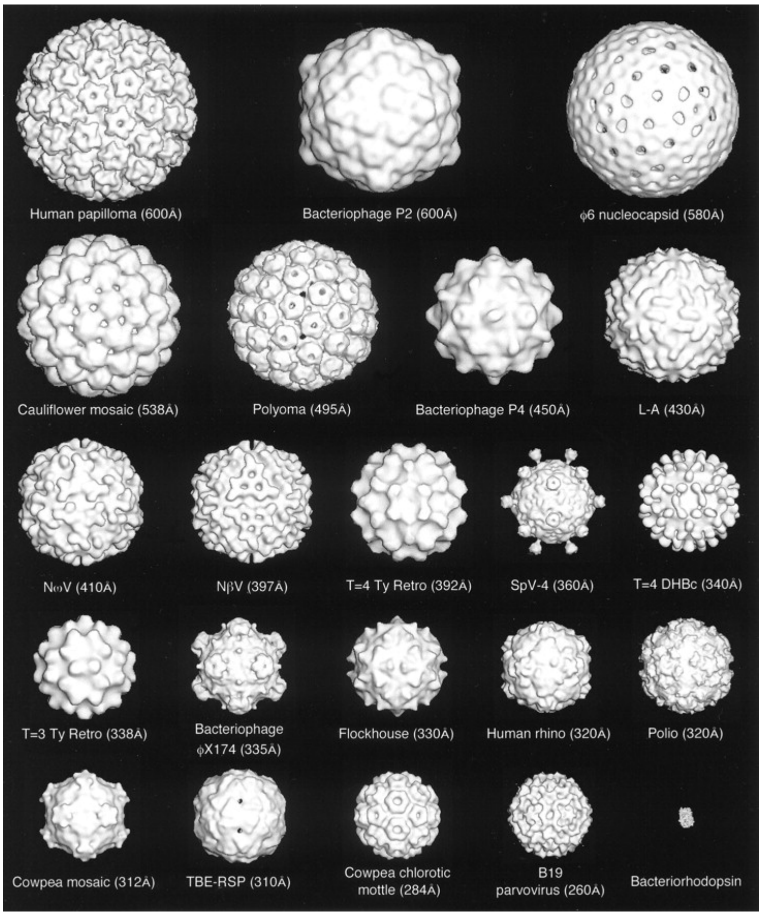
1: Energy. "Self assembly (of a virus) is a process akin to crystallization and is governed by the laws of statistical mechanics. The protein subunits and the nucleic acid chain spontaneously come together to form a simple virus particle because this is their lowest (free) energy state" [17] p. 3. Thus, the symmetries of viruses reflect the spacial symmetry of the physical laws.

The simplest instance of this is seen in the spherically symmetric shapes of small drops of liquids. Spheres "solve" the *isoperimetric problem*: they surround given volume by a surface of minimal area. (The isoperimetric problem goes back to ≈900 BCE and is associated with the name of *Dido* who was, according to ancient Greek and Roman sources, the founder and first Queen of Carthage.)

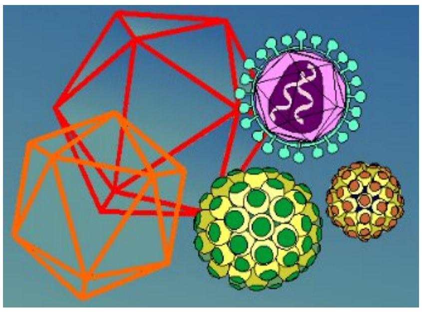
However, not all symmetric problems necessarily have equally symmetric solutions. (Actually, the sphericity of the minimizer to $area/volume^{\frac{2}{3}}$ is not at all obvious; the first complete proof, probably, is < 100 years old.) And the symmetry may look even paradoxical in the case of viruses since the the protein subunits and the nucleic acid chains are not especially symmetric. After all, why should *non-symmetric* (potato-shaped) protein molecules (often) make symmetric crystals?

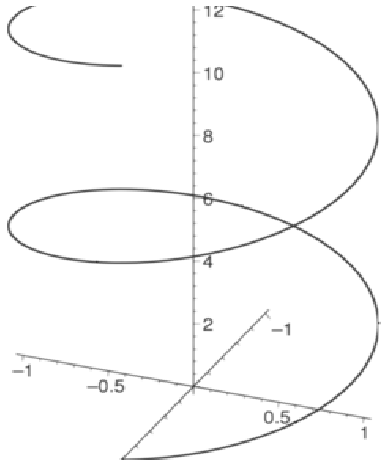
(The first to be crystallized was *haemoglobin* – a rather large protein of ≈ 600 amino acid residues (schematically represented on p.???). The crystals were obtained by Otto Funke (and Karl Reichert?) around 1850 by diluting red blood cells with a solvent followed by slow evaporation.)

The easiest to account for is the *helical symmetry* of molecular assemblies [24], [31], [60]. Suppose that two molecular (sub)units of the same species M preferentially bind by sticking (docking) one to another in a certain way (i.e. the binding energy for a pair of molecules has a unique minimum sufficiently



Human papilloma (600Å)
Bacteriophage P2 (600Å)
φ6 nucleocapsid (580Å)
Cauliflower mosaic (538Å)
Polyoma (495Å)
Bacteriophage P4 (450Å)
L-A (430Å)
NωV (410Å)
NβV (397Å)
T=4 Ty Retro (392Å)
SpV-4 (360Å)
T=4 DHBc (340Å)
T=3 Ty Retro (338Å)
Bacteriophage φX174 (335Å)
Flockhouse (330Å)
Human rhino (320Å)
Polio (320Å)
Cowpea mosaic (312Å)
TBE-RSP (310Å)
Cowpea chlorotic mottle (284Å)
B19 parvovirus (260Å)
Bacteriorhodopsin





separated from other local minima). If $M_1 \vDash M_2$ is a pair of so bound molecules in the Euclidean space, denoted $\mathbb{R}^3$, then there is a (typically unique) *isometric transformation* (rigid motion) T of $\mathbb{R}^3$ moving M_1 to M_2.

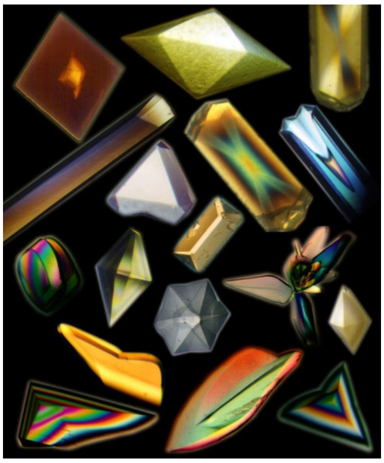
Such a T, by an elementary theorem, is made by a rotation around an axial line L in $\mathbb{R}^3$ followed by a parallel translation along L. If the copies $M_1, M_2 = T(M_1), ..., M_n = T(M_{n-1})$ do no overlap, the chain of n copies of M, written as $M_1 \vDash M_2 \vDash ... \vDash M_n$, makes a helical shape in the space around L which provides a minimum of the binding energy to the ensemble of n molecules. This minimum, even if it is only a local one, has a rather large *attraction basin* (we shall explain it in ???) which make the helical arrangement kinetically quite probable.

Helical symmetries (α-*helices*) are ubiquitous in proteins as was determined by Pauling, Corey, and Branson in 1951 on the basis of the structures of amino acids and the planarity of *peptide* bonds.

(A *polypeptide* is a polymer chain $A_1 \vDash_p A_2 \vDash_p A_3 \vDash_p ...$ made out of small basic unites - *amino acid residues.* There are 20 *standard* amino acids, ranging in size between 11 atoms in *Glycine* and 27 in *Tryptophane*; most (not all) proteins in cells are composed of these. A typical length of a chain is 100-300 residues, reaching > 34000 in *titin* or *connectin*, $C_{132983}H_{211861}N_{36149}O_{40883}S_{69}$ – adhesion template for the assembly of contractile machinery in skeletal muscle cells. A "residue" is what remains of an amino acid after polymerization: the relatively strong covalent *peptide bonds* $\vDash_p$ is formed between a Carbon atom in each (but the last) amino acid molecule in the chain with a Nitrogen atom in the next amino acid with a production of a water molecule.

Immediately upon polymerization, a polypeptide chain, synthesized in the cell by the *ribosomal machinery* "folds" into a specific rather compact shape, called *protein*, held by additional *week* binding forces between residues, such as *hydrogen bonds.* Some proteins are made of several polypeptide chains. For example, *haemoglobin*, which makes about 97% of the red blood cells dry content, is composed of four $\approx$ 150-long subunit chains).

DNA molecules also have helical symmetry. (Three types of DNA double-helices have been found). A helix is composed of two polymer chains held together by hydrogen bonds. (These, usually long, chains, e.g. reaching $\approx 2 \cdot 10^8$ in human chromosomes, are made from four species of *nucleotides*: adenine (A),



guanine (G), cytosine (C) and thymine (T) of 15 ± 1 atoms each.)

Another kind of helix is that of Tobacco Mosaic Virus capsid made from 2130 molecules of coat protein [87].

The "biological helices", are "more symmetric" than the "naked helix" – (almost) every subunit has *more than two* neighbors bound to it. The origin of "bio-helical" and non-helical symmetries can be understood by looking at the 2-dimensional version of the rigid motion T in the 3-space.

A typical isometry T of the plane is a rotation by some angle α around a fixed point, where this α is determined by the "binding angle" of $\vDash$ between M_1 and M_2. If $\alpha = 2\pi/n$ where n is an *integer*, then the n copies of M make a roughly circular shape with the n-fold rotational symmetry.

Thus, for example, not all molecular units M could form a 5-fold symmetric assembly, but if the "$\vDash$-angle" is (planar and) close to $2\pi/5$ and if the $\vDash$-bond is slightly flexible then such assembly (with every copy of M involved into *two* slightly bent $\vDash$-bonds) will be possible.

Returning to the 3-space, if one wants a molecular assembly (e.g a viral shell) V where there are more than two, say 4 neighbors for each copy of M with two different kinds of $\vDash$ bonds and such that V admits say, two symmetries T and T', then one needs to satisfy certain relations between the "$\vDash$-angles" of mutually bound molecules, similar to but more complicated than the $2\pi/n$-condition.

These geometric relations must ensure the algebraic relations between the generators T and T' in the expected symmetry group Γ, where Γ is not given beforehand – it comes along with the self assembly process and may depend on specific kinetics. For example, the symmetry of protein crystals, one of 230 possible crystallographic groups, may depend on the particular condition at which a given protein is being crystallized.

The isometry group Γ itself does not determine the geometry of a Γ-symmetric assembly: specific generators T, T',...of Γ are essential. For example, the helical symmetry is governed by the group $\Gamma = \mathbb{Z}$ of integers. This $\mathbb{Z}$, in "biological helixes", is given by two or more generators, say, by T and T' with the relation $T' = T^n$ with a moderately large n, e.g. $n = 4$ for α-helices, where T is associated with the (strong covalent) *peptide* bonds and T' with (weak) *hydrogen bonds* between amino acid residues in a protein. The Tobacco mosaic virus capsid has $T' = T^{16}$ (?) where, apparently(?) the T-bonds are stronger than T'-bonds.

Alternatively, one can think in terms of the full configuration space $\mathcal{M}$ of molecules M, where there is an action $\mathcal{A}$, or a family of actions, by a group Γ on $\mathcal{M}$. If a (energy) function E on $\mathcal{M}$ is invariant under these actions, then one can show in many cases that the local minima of E on the *subspace* of Γ-*symmetric* configurations also serve as local minima on *all* of $\mathcal{M}$.

For example, $\mathbb{R}^3$ admits a 9-parameter family of actions A of the group $\Gamma = \mathbb{Z}^3$ (where each A is generated by 3 parallel translations) which induce in an obvious way actions $\mathcal{A}$ of Γ on the (infinite dimensional) configuration space $\mathcal{M}$ of molecules in $\mathbb{R}^3$. Since the (mean) binding energy between molecules is invariant under $\mathcal{A}$, the appearance of organic (tri-periodic) crystals (regarded as minima points in the configuration space of molecules) looks less miraculous. (Crystals and crystal growth have been much studied by mathematical physicists but I doubt that a *comprehensive purely mathematical* model incorporating an evaluation of the attraction basin of a crystals and a rigorous quantitative kinetics of crystallization is available.)

In a nutshell, if one is ready to disregard the uncomfortable fact that symmetric equations may have non-symmetric solutions, one might simply say that since the physical world is symmetric, symmetric forms are likely to be functionally as good, if not even better, than non-symmetric ones. For example, the bilateral symmetry of our bodies is good for walking. (Symmetry is persistent in linear systems, e.g. in small oscillation of viral capsids [1], [84].)

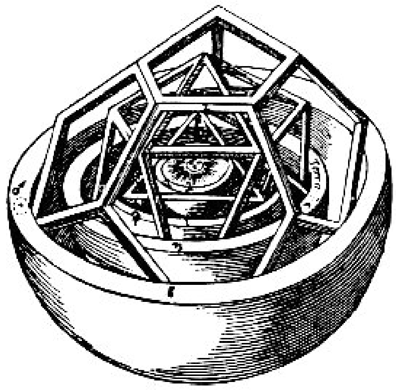
2: Information. The above shows that viral symmetry is plausible but not necessarily *very* probable. The decisive reason for the symmetries of viral shells (capsids) set forth by Crick and Watson was that a virus needed to pack "maximum of genetic information" in a small shell (capsid) which is built of proteins encoded by the viral genes. (The idea that DNA codes for proteins was in the air since the reconstruction of the DNA double helix structure by Crick and Watson in 1953 and Gamov's 1954 suggestion that each of 20 amino acids must be coded by a triplet of nucleotides, since $n = 3$ is the minimal solution to the inequality $4^n \geq 20$, where 4 is the number of different species of nucleotides in DNA.)

Indeed, if a virus uses n genes in its DNA (or RNA) for the shell proteins, say, each codes for m copies of identical proteins molecules, then the resulting viral shell can contain DNA of size $\sim (nm)^{\frac{3}{2}}$. If m is large, this allows smaller n which is advantages to the virus. (A small virus replicates faster and more copies of it can invade the same cell.)

The above energy argument implies that the presence of equal copies of protein molecules make symmetric assembles quite likely if one properly adjust the "$\vDash$-angles"

Now, an evolutionary factor enters the game: a symmetric form can be specified by fewer parameters than a functionally comparable non-symmetric one. For example, a non-symmetric assembly of molecules may have many different "$\vDash$-angles" all of which need be somehow encoded by the viral DNA (or RNA) while a symmetric form has many of these "angles" mutually equal. This simplifies Nature's task since it selects from a smaller pool of competing possibilities.

(The presence of *many identical* copies of *large heterogeneous* "units", e.g. heteropolymeric molecules, is the hallmark of life. These are produced by some process of *controlled amplification* – another characteristic feature of living sys-



tems. An instance of this is templating mRNA from DNA followed by translation from mRNA to proteins. On the other hand, ignition of combustion or of a nuclear chain reaction are examples of uncontrolled amplification.)

Abstractly, one minimizes some combination of the total binding energy between protein molecules and the "information/selection cost" of DNA encoding for these molecules, but a mathematical rendition of Crick-Watson idea is still pending – no one of "isoperimetric animals" cultivated by geometers for the last 3 000 years resembles icosahedral viruses. (There is infinitely more to virus structures than all of the above, see Virus-Wikipedia for a brief account of virology and for references.)

In the end of the day, the symmetry of viruses depends on the structural constrains imposed by the geometry of the *physical space* which allows the existence of such improbable objects as icosahedra (See [47] for a continuation of our discussion on proteins.)

Similarly, we expect that the constrains of the *mathematical space* (which we have not define yet) where ergosystems reside will have a strong structural imprint on a possible architecture of the ergobrain. For example, we want to understand what in the (ergo)brain is responsible for the unreasonable power of our visual system which allowed a discovery of icosahedra by humans (at least) as early as -400.

Lamentably, we do not know yet what kind of structure to look for: one can not take something, however attractive it can be, just because it happened to be close at hand.

In 1595 Johannes Kepler observed that each of the five Platonic solids (regular polyhedra) could be uniquely inscribed and circumscribed by spherical orbs; nesting these solids, each encased in a sphere, within one another would produce six layers, corresponding to the six known planets: Mercury, Venus, Earth, Mars, Jupiter, and Saturn. He believed at some point this was the fundamental law of nature.

This may look naive from the 21th century perspective, but the essence of Kepler's idea is very much alive to-day: the *fundamental* laws of physics *are* governed by fundamental (probably, still unknown to us) symmetries. However, we do not believe anymore that there is anything fundamental in *our* planetary system (this idea died out by end of the 19th century) nor the Platonic bodies are considered fundamental objects in the modern group theory. (See [68] for an overview of symmetries in nature.)

Much of mathematics the reader will meet before we turn to ergosystems may appear as remote from brain's (ergo)functions as Platonic bodies from planetary motions: mathematicians do not forge their tools for anybody's applications. What is instructive, however, is how one makes the first steps in shaping new mathematical concepts and theories. This is similar, we believe, to how the ergobrain operates in other situations; we shall try elucidate a few pathways along which "raw intuitive" ideas undergo transformations into "final" mathematical products.

1.4 Animals Play, Animals Learn, Animals Dance, Animals Talk.

Play is, as we all have chance to observe, is the main ergo-interesting activity of human babies, kittens and puppies but students of animal behavior may say [7] that "like other beautiful things play is ephemeral" and that:

Play is correlated with the development of the neocortex in mammals. (also of cerebellum? and of pallium in birds?). Many (most?) species of young mammals are known to play as well as some (many?) species of birds (e.g. crows and ravens) and, possibly, some turtles. (There are claims that ants were observed to play.)

Playfulness retained into adulthood, e.g. in humans and dogs, goes along with other neonatal characteristics. Some adults animals in the wild also paly, e.g. *dholes* – red Asian dogs *http://www.youtube.com/watch?v=ejnuprap_RY.*

Dholes are smaller than wolves, around 15kg of weight and quite agile, e.g. jumping more than 2m high. They are more social than wolves and, unlike wolves, have little of a dominance hierarchy. Preparing for a hunt, they go through elaborate rituals of nuzzling, body rubbing and homo and heterosexual mounting. Though fearful of humans, dhole packs are bold enough to attack large and dangerous animals such as wild boar, water buffalo, and even tigers. Travelling through thick brush, dholes coordinate their movements by whistles the mechanism of which remains unknown. Unlike most social canids, dholes let their pups eat first at a kill. (Wikipedia)

Native people killed dholes by poisoning, snaring, shooting and clubbing pups at den sites to protect livestock while European sport hunters killed for fun. Read this:

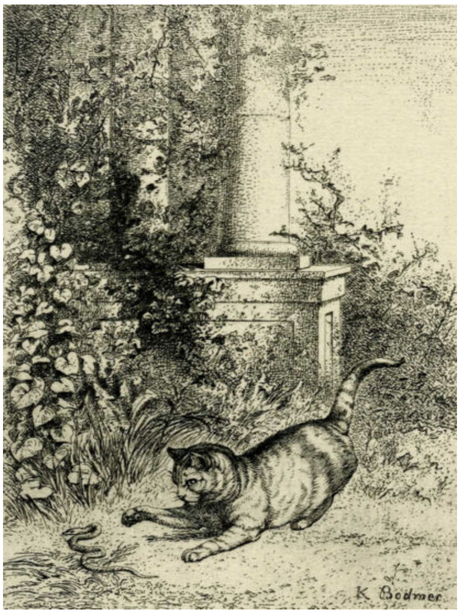
"... no effort, fair or foul, should be spared to destroy these pests of the jungle..." that have "not a single redeeming feature".

This is about *dholes*, not about sport hunters, and written as recently as 1949 by somebody who called himself "clean-footed naturalist".

http://www.cuon.net/dholes/danger.htm.

(Rudyard Kipling depicts the ancestral program running the minds of such "naturalists" in The Jungle Book: "... he [Mowgli] had seen the fearless dholes sleeping and playing and scratching themselves in the little hollows and tussocks that they use for lairs. He despised and hated them because they did not smell like the Free People, because they did not live in caves, and, above all, because they had hair between their toes while he and his friends were clean-footed.")

It is estimated that only 2500 dholes remain in the wild.



Play involves behavior patterns adapted from their usual contexts. But also there is the specific communication: "This is play" between animals.

Apparently, animals play for "fun" when they are not preoccupied with feeding, fleeing, fighting or mating, but "fun" is difficult to recognize in animals that are separated from us phylogenetically (actually, it is not always clear what behavior can be classified as "play").

There are two (pragmatic ego-oriented) competing views on animals, including humans, play:

(1) Play is a preparation for the future (Septics say "a preparation for more play")

(2) Play is a legacy from the past.

There is no convincing evolutionary explanation for the play. Our non-professional attitude is that many patterns of play are reflections of some facets in the mental architecture of the ergobrain. These patterns were not individually targeted by selection: their benefit, if any, was not deserving the attention of Evolution. Possibly, play even came about *despite* not because of selection,

(Eventually, selection may win out and populate Earth exclusively with bacteria which would have no risk-prone inclination for play. Probably, on can prove, almost mathematically, that the evolutionary/quasi-termodinamically most stable/probable state of the surface chemistry of an Earth-like planet makes a biosphere composed of organisms with 10^9-10^{15} atoms. Conceivably, 99% of the planets would carry such biospheres, provided the "ensemble of planets" is super-cosmically large, something like $10^{10^{10}}$ planets.)

If benefits of play remain questionable for animals, play *had* brought a few nice unexpected things to humanity, such as electricity, radio and penicillin. This is how Alexander Fleming put it in his 1945 speech in Louvain:

"I play with microbes. There are, of course, many rules in this play....but...it is very pleasant to break the rules... . " (Taken from [66], p. 246.)

Now listen to Howard Walter Florey, whose titanic efforts to extract, purify, test and produce sufficient quantities of penicillin together with Ernst Chain, Norman Heatley and their coworkers in 1938-1941 in Oxford had brought penicillin to the therapeutic use by mid-40s. (Penicillin had saved close to 100 million human lives since 1945.) Florey, said in an interview:

"This was an interesting scientific exercise, and because it was of some use in medicine is very gratifying, but this was not the reason that we started working on it."

Finally, La Touche – the mycologist at St Mary's Hospital working in the room below Fleming from where the spores of a rare strain of Penicillium notatum had, probably, originated and, between 27 July and 6 August of 1928, contaminated a Stafilococci plate in Fleming's lab – can hardly be suspected of anything but playing with fungi.

Kanzi. (Wikipedia) "As an infant, Kanzi (bonobo) accompanied his adoptive mother Matata to sessions where she was taught language through keyboard lexigrams, but showed little interest in the lessons.

When one day, while Matata was away, Kanzi began competently using the lexigrams, becoming not only the first observed ape to have learned aspects of language naturalistically rather than through direct training. Within a short time, Kanzi had mastered the ten words that researchers had been struggling to teach his adoptive mother, and he has since learned more than two hundred more. When he hears a spoken word (through headphones, to filter out nonverbal clues), he points to the correct lexigram." (Adult Kanzi can flake Oldowan style cutting knives. He understands how to beat the arcade game Pac-Man.)

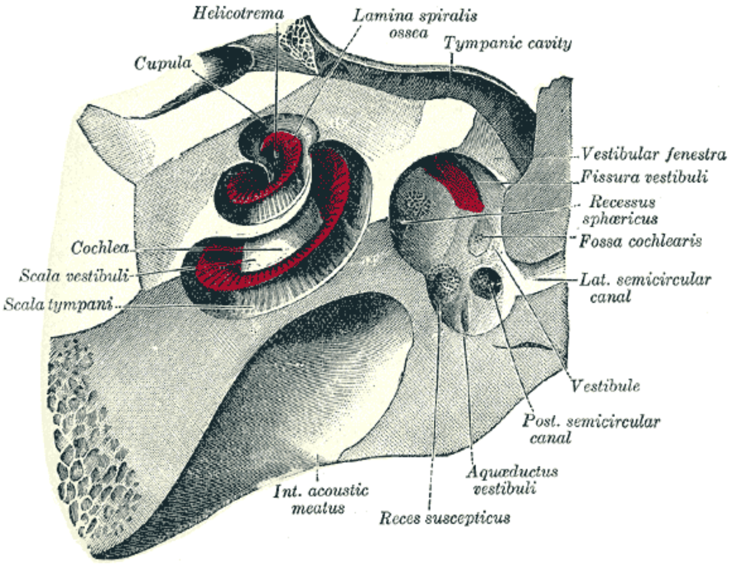
The obvious (for some and paradoxical for others) advantage of Kanzi over his mother was that he *did not understand* the teaching instructions, he *had not developed* the taste for (internal and external) emotional rewards and *had not learned* how to obey.

It may seem there was no essential stimulus, no reason for Kanzi to learn, but *self propelled (or free) structure learning* is the game babies play.

In fact, "ego-stimuli" may inhibit learning rather than stimulate it as the following story, shows (http://www.intropsych.com).

"Zoo-psychologist Sarah Boysen found that two of her chimpanzee subjects, Sarah and Sheba, were both capable of learning such concepts as "more than" and "less than" easily.

Boysen then used chimpanzee's favorite gumdrops as stimuli. The chim-



panzee was presented with two plates of gumdrops. One had more on it, the other had less. While the other chimpanzee watched, the chimpanzee being tested was asked to point to one of the plates. Whichever plate it pointed to was given to the other chimpanzee.

In this situation, the chimp doing the pointing should have learned to point to the plate with fewer gumdrops on it, in order to have that plate given to the other chimpanzee and in order to get the plate with more gumdrops for itself. Instead, the chimpanzee insisted on pointing to the plate that had more gumdrops, even though these went to the other chimpanzee. Chimpanzees seemed to know they were making a mistake but they could not stop themselves.

Then Boyson replaced gumdrops with plastic poker chips. Now the chimpanzees had no trouble with the task. They pointed to the plate with fewer poker chips on it. This meant the plate with fewer gumdrops went to the other chimp, and the chimp that did the pointing got the larger number of gumdrops."

It seems likely that similar mental blocks, call them *ego-protective walls* are ubiques in humans as well as in animals (ego)-minds, where, e.g. the reward/punishment stimuli inhibit structure learning by a student. At the same time, similar "walls" in a teacher's ego-mind, which was evolutionary "designed" for "listen to me", "obey me", "please me" and "conform to me" rather than for promoting free structure learning, can nullify the best efforts of the kindest of teachers blind to this.

In fact it is amazing that any teaching is possible at all: the structure analyzing ability of a one year old chimpanzee is, conceivably, by an order of magnitude greater than that of the conscious mind of the brightest homo sapience.

Imagine, you see on a computer screen what a baby brain "sees": a "throbbing streaming crowd of electrified shifting points" encoding, in an incomprehensible manner, a certain never seen before, not even imaginable, "reality". Would you reconstruct anything of this "reality"? Would you be able to make such concepts as "shadow", "roundness", "squareness"? Could you extract any "meaning" from a Fourier-like transform of the sound wave the brain auditory system receives, even if you know a bit more of Fourier transform and of the anatomy of the inner ear than Kanzi does?

The infant's brain God-like ability of making a coherent world out of apparent chaos is lost by "mature" minds. One can not even recognize 2-dimensional images by looking at graphical representations of the illumination levels, which is a much easier problem. What a baby chimpanzee's brain does is more "abstract" and difficult than the solution of the Fermat's Last Theorem recently found by mathematicians.

(For more than 50 years, apparently having been oblivious to acoustical limitations of ape's larynx and a likely associated disability of ape's auditory system to identify patterns making human speech, psychologists kept attempting to teach apes to talk, see "Animals and language" in [64]. These disabilities, have nothing to do with ape's ergobrain potential for recognizing/generating syntactic structures but experimental evaluation of this potential is difficult due to our limited understanding of the ergo-structures involved.)

Learning by baby apes is, possibly, the most structurally complicated process in animals brains, but there are other amazing patterns in sexual/social behavior of animals which can not be blatantly delegated to "cleverness of evolution" and which beg for a structural explanation. Below are a few examples.

"The male Costa's Hummingbird (who weights 2.5– 3.5 g) courtship display is a spirited series of swoops and arcing dives, carefully utilizing a proper angle to the sun to show off his violet plumage to impress prospective mates. Each high-speed dive will also pass within inches of the female, perched on a nearby branch, which will be accented by a high-pitched shriek". (Wikipedia)

"Males bowerbirds build a bower to attract mates. Satin Bowerbird (about 200 g in weight), the best known and well documented of all the bowerbirds in Australia, builds an avenue type bower made of two walls of vertically placed sticks. In and around the bower the male places a variety of brightly colored objects he has collected. These objects usually different among each species may include hundreds of shells, leaves, flowers, feathers, stones, berries, and even discarded plastic items, coins, nails, rifle shells, or pieces of glass. The males spend hours arranging this collection.

Bowers within a species share a general form but do show significant variation, and the collection of objects reflects the biases of males of each species and its ability to procure items from the habitat often stealing them from neighboring bowers. Several studies of different species have shown that colors of decorations males use on their bowers match the preferences of females.

In addition, many species of bowerbird are superb vocal mimics. Macgregor's Bowerbird, for example, has been observed imitating pigs, waterfalls, and human chatter. Satin bowerbirds commonly mimic other local species as part of their courtship display". (Wikipedia)

"Dancing is one of the most obvious displays by any social bird, and all species of cranes do it. A dance involves stiff-legged marching, running and leaping into the air with spread and beating wings, bowing, pirouetting, stopping and starting and tossing twigs into the air."

$http://www.news.cornell.edu$
$/chronicle/04/9.16.04/crane_dance.html$

Some highly intelligent birds, like the crow, are only able to mimic a few

words and phrases, while some parakeets have been observed to have a vocabulary of almost two thousand words.

The African Grey Parrots (about 500g of weight) are noted for their cognitive abilities.

Alex had a vocabulary of about 100 words. In 2005, World Science reported that Alex understood the concept of zero.

Prudle held the Guinness world record for bird with biggest vocabulary for many years with a documented vocabulary of 800 words.

N'kisi, as of January 2004, had a documented vocabulary of 950 words and, according to the loving owner, shows signs of a sense of humor.

Einstein is able to recreate sounds as well as voice, for example, she can make the sound of a laser beam and an evil laugh."(Wiki)

Parrots can be taught to speak without much stimuli: if a mirror is placed between the parrot and the trainer, the parrot, seeing his own reflection in the mirror, fancies another parrot is speaking, and imitates all that is said by the trainer behind the mirror. The teaching goes well if your bird is happy, relaxed and... young.

1.5 Learning to Talk.

All neurologically healthy (and even partially impaired) children spontaneously learn languages without any reward/punishment reinforcement [32], which appears as miraculous and inexplicable as strong inclinations of some children to play chess, perform/compose music, to study science and to do mathematics.

A child language acquisition works fine *without* supervision. In fact, a pressure by a teacher inhibits language development in children according to studies by W. Labov and by S. Phillips, see p. 299 in [64]. (The unhappy supposition is that this applies to learning mathematics by children as well.)

Due to the complexity of the problem and the lack of study it remains unclear which kind of teaching practices promote and which suppress the learning process by children, but it is painful for a teacher to accept that his/her intervention may be harmful rather than beneficial.

(This is similar to the dilemma having been faced by medical doctors for centuries. For example, it was established by Almroth Edward Wright, Alexander Fleming and their coworkers during the First World War that antiseptics applied to fresh wounds were more likely to kill phagocytes than bacteria, such as ordinary streptococci and staphylococci as well as deadlier anaerobic tetanus bacilli and Clostridium perfringens causing gas gangrene [66] ch. 9. But it took a couple of decades before the wound treatment suggested by Wright became a standard surgical procedure.)

Noam Chomsky, Eric Lenneberg and their followers believe (see ch.12 in [48]) that children have an evolutionary installed "innate universal grammar" in their heads that facilitates and constrains language learning. (Apparently, [74] Chomsky himself does not adhere to this idea anymore. See [74] for a roboticist's perspective on language generating mechanisms and other emergent cognitive phenomena.) In fact, learning a "general language" even with a simple grammar may be virtually impossible [43].

On the other hand, besides languages, children easily learns chess and bonobos display Pac-Man playing ability. Even more inexplicably, young students can learn mathematical arguments which, if logically expounded in full detail, would cover tens (hundreds?) thousand pages, e.g. the proof of Fermat's Last Theorem [23].

(The results of recent neurological studies of how we learn to *read* [33] can be interpreted in favour as well as against Chomsky-Lenneberg thesis.)

We shall explain later on how the universal structure learning mechanism accounts for the language acquisition along with chess (regarded as a dialog between the players) and mathematics (where ergobrain plays with itself [11]) with agreement with the point of view currently accepted by many psychologists and computer scientists [55].

Besides the ordinary spoken languages there are several types of less common ones.

Whistled languages (Wikipedia) are systems of communication which allow fluent whistlers to transmit and comprehend messages over several km distances, with the whistling being either tone or articulation based.

Whistled languages function by varying the frequency of a simple wave-form and convey phonemic information through tone, length, and, to a lesser extent, stress, where many phonemic distinctions of the spoken language are lost [14].

The *Silbo* on the island of La Gomera in the Canary Islands, based on Spanish, is one of the best-studied whistled languages. The number of distinctive sounds or phonemes in this languages is a matter of disagreement, varying according to the researcher from two to five vowels and four to nine consonants. This variation may reflect differences in speakers' abilities as well as in the methods used to elicit contrasts.

The language of 200-300 hunter-gatherer *Pirahã* people, living in a few villages on the banks of the river Maici deep in the Amazon rainforest in Brazil, can be whistled, hummed, or encoded in music with the meaning conveyed through variations in pitch, stress, and rhythm.

The Pirahã language, unrelated to any other living language, has been studied by Keren and Dan Everett who lived with Pirahã people from 1978 to 1983 and from 1999 to 2002. Many unusual, often controversial, features of Pirahã [25], identified/claimed by Everetts, have brought this language to the focus of an exceptional interest among linguists and anthropologists.

Whistled languages tend to disappear in the face of advancing telecommunications systems. For example, in the Greek village of Antia, only few whistlers remain (2005) but in 1982 the entire population knew how to whistle their speech.

Sign languages, which commonly develop in deaf communities, have a high *non-sequential* component: many "phonemes" are produced simultaneously via visually transmitted patterns combining shapes, orientations and movements of the hands, arms and body as well as facial expressions.

Before the 1970s, deaf people in Nicaragua were largely isolated from each other. In 1980, a school for adolescent deaf children was opened and by 1983 there were over 400 students enrolled. Initially, the language program emphasized spoken Spanish and lip-reading; the use of signs by teachers was limited to fingerspelling. The program achieved little success, with most students failing to grasp the concept of Spanish words (Wikipedia).

But while the children were *linguistically disconnected* from their teachers they were communicating by combining gestures and elements of their home-sign systems, a pidgin-like form, and then a creole-like language rapidly emerged.

Then the young children had taken the pidgin-like form of the older children to a higher level of complexity, with verb agreement and other conventions of grammar. (Some linguists questions assertions that the language has emerged entirely without outside influence.)

The communication problem seems insurmountable for deaf-blind people; they succeed, nevertheless. Different systems are in use, one is *Tadoma* which is tactile lip-reading (or tactile speechreading). The Tadoma user, feels the vibrations of the throat and face and jaw positions of the speaker as he/she speaks. Unfortunately, this requires years of training and practice, and can be slow, although highly-skilled Tadoma users can comprehend speech at near listening rates,

Below are two pieces of poetry written by deaf-blind people.

From "A Chant of Darkness" by Helen Keller
http://www.deafblind
.com/hkchant.html

In the realms of wonderment where I dwell
I explore life with my hands;
I recognize, and am happy;
My fingers are ever athirst for the earth,
And drink up its wonders with delight,
Draw out earth's dear delights;
My feet are charged with murmur,
The throb, of all things that grow.

This is touch, this quivering,
This flame, this ether,
This glad rush of blood,
This daylight in my heart,
This glow of sympathy in my palms!
Thou blind, loving, all-prying touch,
Thou openest the book of life to me.

****

“My Hands“ By Amanda Stine.

My hands are . . .
My Ears, My Eyes, My Voice . . .
My Heart.
They express my desires, my needs
They are the light
that guides me through the darkness

They are free now
No longer bound
to a hearing-sighted world
They are free
They gently guide me

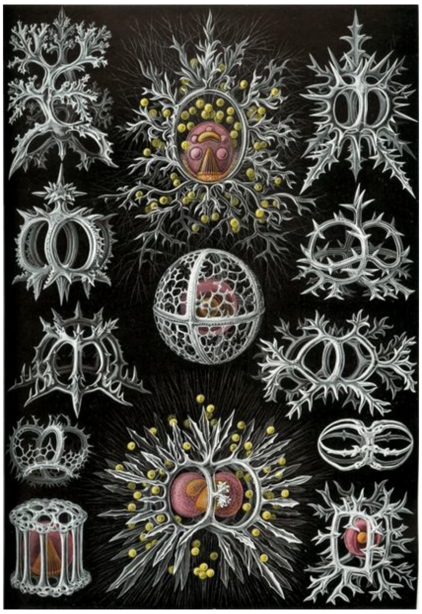
With my hands I sing
Sing loud enough for the deaf to hear
Sing bright enough for the blind to see

They are my freedom
from a dark silent world
They are my window to life
Through them I can truly see and hear

I can experience the sun
against the blue sky
The joy of music and laughter
The softness of a gentle rain
The roughness of a dog’s tongue

They are my key to the world
My Ears, My Eyes, My voice
My Heart

They are me



http : //www.deafblind.
com/myhands.html

1.6 Interesting, Amusing, Amazing, Funny and Beautiful.

Ergo-moods, such as “amusement” and “amazement”, are indicators/signatures (as well as dynamic components) of the working of the ergobrain. It, being “unpractical and illogical”, prefers solving “useless” crossword puzzles to filling “useful” tax forms. Our visual system is *amused* by optical illusions, *amazed* by tricks of magicians, *fascinated* by performance of gymnasts. Our auditory system is *enchanted* by music. Our olfactory system is attracted by exotic perfumes. Our gustatory system is hungry for strange and often dangerously bitter foods. Our motor/somatosensory system plays with our bodies making us dance, walk on our hands, perform giant swings on the high bar, juggle several unhandy objects in the air, climb deadly rocks risking our lives, play tennis, etc.

Also the *aesthetic perception* is very much of ”ergo” albeit the initial source of it can be seen in the context of the sexual selection. But the kaleidoscopic symmetry of peacock’s tails and ornamental designs, the architecture of plants, animals and cathedrals, music, poetry – all that with no ”survival/reproduction“ tag on them, trigger in us a feeling similar to what is induced by images of the opposite sex. (The similarity/connection between the two kinds of aesthetic feelings is so strong that a structurally beautiful poem transmits emotions experienced by a poet in the thralls of love to the mind of the reader.)

The seduction of the the structural beauty is sometimes irresistible: it overrides the pragmaticity dictum of evolution and lures us to chess, to the game of Go to arts, to music and to the ultimate human game: the mad pursuit of beautiful structures in science and mathematics.

We shall argue later on that funny in jokes, harmony in quasi-periodic tunes

and beauty in symmetric patterns are generated by similar ergo-recognitions mechanisms.

Pig Watching. A man passing a small apple orchard saw in amazement as the farmer was lifting one pig after another on a complicated mechanical contraption up to an apple tree waiting patiently while the pigs ate slowly apples from the tree.

The man stood there fascinated for half an hour watching these pigs. His curiosity eventually won out, and he asked the farmer:

—Why are you doing this?-

The farmer said— Apples improve pigs' digestion and add flavor to their meat.

– But doesn't all that take a hell of a lot of time?

The farmer gave the man a long stare and responded –

– What's time to a pig?

Question to the Reader. Was the farmer mocking the city man who waisted his time watching pigs or was it an honest question?—The farmer, possibly, figured out that the watcher was a zoo-psychologist who could satisfy farmer's curiosity on how pigs perceived time. (Time is different for different systems. For example, a mathematician's time, as we shall see later on, is different from a pig's time.)

1.7 Mathematics Supernovas.

$$\sqrt{1+2\sqrt{1+3\sqrt{1+4\sqrt{1+5\sqrt{1+\cdots}}}}}=3$$

The beauty of this identity is apparent to everybody who knows what $\sqrt{1+2\sqrt{1+...}}$ signifies. This was one of the first formulas published by the Indian mathematician Srinivasa Ramanujan (1887-1920). The proof is very hard to guess but it can be written in a few lines explicable to a high school student.

But not every mathematician can penetrate the depth of the following *Ramanujan mysterious formula for* $\pi = 3.1416....$. (Roughly, I think, less than one out of ten of my mathematicians friends and colleagues is up to this, and it is not myself.)

$$\frac{1}{\pi} = \frac{2\sqrt{2}}{9801} \sum_{k=0}^{\infty} \frac{(4k)!(1103 + 26390k)}{(k!)^4 396^{4k}} = \frac{2\sqrt{2}}{9801} \left(1103 + \frac{24 \cdot 27493}{396^4} + ...\right)$$

The formula miraculously equates something as simple and familiar as $1/\pi$ with the infinite sum of the illogically complicated terms (where $k! = 1 \cdot 2 \cdot 3 \cdot ... \cdot k$ with the convention $0! = 1$) on the right hand side. (Amazingly, this gives a fast algorithm for computing the decimals of π.)

When he was 16, Ramanujan came across a book compiling 5000 theorems and formulas; this introduced Ramanujan to mathematics. Ramanujan did not complete the formal education: he was too intent on studying mathematics. Without ever getting a degree he continued to pursue independent research, often near the point of starvation. In 1914, he was invited by Godfrey Hardy, the leading English mathematician, to Cambridge. Ramanujan returned to India in 1919 where he died the next year.

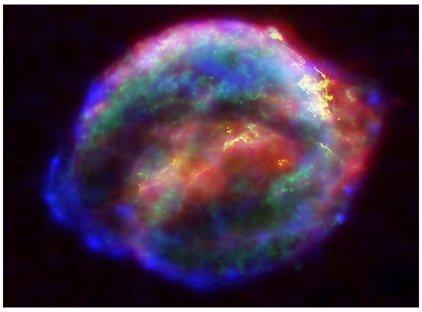
Ramanujan recorded his results, formulas and statements of theorems without proofs in four notebooks. The results in his notebooks inspired numerous mathematicians trying to prove what he had found. The fourth notebook–a bunch of loose pages– the so-called "lost notebook" with about 650 of Ramanujan's formulas, most of them new, was rediscovered in 1976 by George Andrews in the Library at Trinity College. George Andrews and Bruce Berndton collaborated on publishing the proofs for Ramanujan's formulas included in the lost notebook; two out of the four expected volumes of their work are in print.

A supernova is an explosion of a massive (several times heavier than the sun) star. It may shine with the brightness of 10 billion suns. Only a dozen of supernovas were observed during the recorded history (none since the invention of the telescope) but, probably, there are several billion of stars in our galaxy which will turn supernova when their time comes (most astronomers believe so) but, apparently, most supernova's explosions are obscured from us by interstellar matter.

Supernovas in mathematics are seen as rarely as in the sky; yet all human on earth, are as similar to each other in the basic neural abilities (walking, handling, speaking...) as the stars in a single supernovas' family.

What makes a star (of a given mass) a supernova is (essentially) the value of a single parameter, the age of the star. What made Ramanujan, I believe, is a minor (on the brain scale) increment of something like the "conductivity parameter" of the communication line from his (ergo)brain to his his conscious mind or a decrement in translucency of the "brain/mind window".

Rare mental abilities could not have been evolutionary selected for and structurally complex functional features (be they anatomical or mental) can not come by an accident. It follows that the hidden mental power of everybody's (ergo)brain, not only of Ramanujan's brain, must be orders of magnitude greater than that of the (ego)mind. (The development of the brain is, up to large extent, a random process, where only its general outline is genetically



programmed. This may allow rare fluctuations of some average “connectedness numbers” which can be further amplified by “Hebbian synaptic learning”; yet, this, probably, does not essentially change what we say.)

The legacy of evolution keeps this ”ergo-power” contained by the ego-protective wall: a hunter-gatherer whose ergobrain had overrun his/her pragmatic egomind did not survive long enough to pass on his/her genes.

Also, the ability of the (ergo)brain to do mathematics which is so far removed from the mundane activities of the (ego)mind is our strongest evidence that much of the ergobrain is run by some simple universal principles: a specialized and/or complicated system, besides being evolutionary unfeasible, could hardly perform several widely different functions. We shall try to identify some of these principles where we use the experience of scientists and mathematicians in recognizing and building structures.

Ego and ergo often follow colliding courses when they approach something extraordinary. Here is an instance of this.

Horse Talking. A man lost in the fields tries to find his way.

Hi – he hears behind him.

He turns around and, astounded, faces a horse.

– You can speak English! – the man exclaims.

I also speak French—answers the horse. Let’s go to my farm and talk on the way.

While they walked the horse kept chatting in English and French about how she had studied at Harvard from where she received her PhD in French Literature. When they came to the farm, the man approached the farmer excitedly:

—Your horse...—

The farmer did not let him finish.

– I know damn well what she’s told you, this mare is no good for anything, a liar at that. She has never been close to Harvard!

Yes, it’s appalling – the man pointed his finger accusingly – I have been teaching French Literature at Harvard for years but this horse has never shown in my class.

For a similar reason, many biologists, including Alfred Russel Wallace, were appalled by Mendel’s ideas when de Vries, Correns, and Tschermak rediscovered Mendelian heredity principles at the turn of the 20th century. In fact, Mendel’s theory and the *Hardy-Weinberg principle* (undoubtedly known to Mendel) imply that the ”artificial selection”, i.e. selective breeding, could not serve as a standard model for the natural selection in-so-far as Death and Extinction, unlike human breeders, allowed *random* mating of their livestock.

This infuriated "classical Darwinists". Those in the Soviet Russia around Lysenko (often accused of Lamarckism quite unjustly to Lamarck) rejected "anti-materialistic Mendelian algebra" as an idealistic abstraction which, they argued, does not, for example, help farmers to raise the yield of milk, since nutritionally minded cows are inept at algebra.

The *Hardy-Weinberg principle*, probably, observed by an American biologist William E. Castle in 1903 for the first time since Mendel (Mendel published his work in 1866 and died in 1884), and fully articulated by the famous English mathematician G. H. Hardy and independently by a German physician Wilhelm Weinberg in 1908, is the following, somewhat counter intuitive, algebraic identity (customary stated for symmetric 2×2-matrices).

Let $M = (m_{ij})$ be a (normalized probability mating) matrix with $m_{ij} \geq 0$ and $\sum_{ij} m_{ij} = 1$. Let $M' = (m'_{ij})$ be obtained by substituting each (i,j)-entry from M by the product of the two numbers: the sum of the entries in the i-row in M and the sum of the entries in the j-column, i.e.

$$m'_{ij} = \left(\sum_j m_{ij}\right) \cdot \left(\sum_i m_{ij}\right),$$

and let

$$M^{new} = \frac{M'}{\sum_{ij} m'_{ij}}, \text{ i.e. } (m^{new}_{ij}) = \left(\frac{m'_{ij}}{\sum_{ij} m'_{ij}}\right).$$

Then

$$(M^{new})^{new} = M^{new},$$

where, observe, the matrix M^{new} may be vastly different from the original M.

This implies that the distribution of a certain phenotypic feature (controlled by a single gene) among chidren may be drastically different from that for a population of parents; yet, this distribution remains constant for the following generations: (illusory) "selection" terminates at the second round of random mating. (See [46] for a mathematical overview of the Mendelian "new generation" dynamics.)

One may speculate that Darwin himself who, like Mendel (and unlike Lysenko with his cows), had a fine feeling for numbers (but, unlike Mendel, was not on friendly terms with the multiplication table [77]), would realize that Mendel's experiments and his logical conclusions had not contradicted to the *spirit* of the "evolution by selection" idea but, on the contrary, could have provided a scientific foundation to the *natural* selection theory with gene *mutations* instead of recombinations.

1.8 Ergoworld, Ergothinking and Mathematics.

Different aspects/fragments of the world have different level of structurality and, accordingly, there are different "parts" of the brain/mind which reflect such fragments. It has so happened that the structural aspects are mostly taken care of by the ergobrain and loose structures are in the domain of the egomind. These "reflections" are the worlds in their own right, call them ergoworld and egoworld, which, besides being different in what they reflect, also differ in their "how" and "why".

The *ego-processes* which create the egoworld and performed, controlled and recorded by the egomind, center around generations of actions and concepts which (more or less) directly serve the goals that are set by the survival/reproduction needs of an organism; particular patterns of egomind were specifically targeted by the evolution/selection mechanism. Many (most?) ego-processes are perceived by retrospection and/or are directly observed in the behavior of human and animals. Egomind is “real”, large and “structurally shallow”.

The ergobrain, unlike the egomind, is a structural entity, which, conjecturally, underlies deeper mental processes in humans and higher animals; these are not accessible either to retrospection or to observations of behavior of people and/or animals. This makes the ergobrain difficult (but not impossible) for an experimental psychologist to study. (“Folk psychology”, “common sense”, ”conventional wisdom”, ”psychoanalysis” tell you as much about non-trivial processes in the mind as astrology does about the synthesis of heavy atomic nuclei in supernovas.)

The ergobrain and the egomind are autonomous entities, although they communicate. In young children, human and animals, the two, probably, are not much separated, but as the egomind (”personality”, in the ego-language) develops it becomes protected from the ergobrain by a kind of a wall. This makes most of ergobrain's activity invisible.

Seeing through this wall is an essential intellectual difficulty in modeling the ergobrain and ergosystems in general. Our ergobrains are entrenched into services of our ego-minds and it is hard to separate the ”real ergo-staff” from the ego-imprints. We attempt to identify and erase these imprints by *every* imaginable means; the pertinacious resistance of our ego-minds, both of the reader and of the writer of these lines, makes this task difficult.

For example, it is hard to mentally switch from the common ideas of ”reality” and its ”meaning” – two stereotyped ”survival ego-programs” installed into you by the evolutionary selection process, to the ”reality” of the ergoworld and to the structural ”meaning” of combinatorics of ”networks of ergo-ideas” in the ergosystems.

(Our emphasis on the ”ergo-reality” should not be confused with the negation of ”physical reality” by solipsists and, more recently, by cosmological biocentrists.

In fact, solipsism, after centuries of unsuccessful attempts, has been eventually brought to the firm ground by Terry Pratchett in his ”Soul Music” of *Discworld*:

”... horses won't walk backward voluntarily, because what they can not see does not exist.”

This confirms what solipsists were predicting all along, Pratchett's presupposition that the equidae, i.e. perissodactyl ungulate mammals, of Discworld possess no recognizable visual organs on their hindquarters notwithstanding.)

The difficulty in modeling the ergoworld is compounded by the lack of a general mental mechanism for inventing new kind of structures: our own ergobrain is severely limited in its structure creative ability. The main source of ideas for designing ”ergo-architectures” comes from mathematics.

The ego-wall also makes ergo-processes hard to study experimentally. For example, if a psychologist or a neurophysiologist give a mathematical task (above the multiplication table level) to a subject, the dominant difficulty of the subject is breaking through his/her ego-ergo wall; all the observer will see is a ”hole in the wall" – the trace of this breaking process.

And when the subject crosses the wall and enters the internal ergoworld, he/she becomes unable to (ego)communicate, as every scientist, artist or mathematician knows. (Composing music and filling-in tax forms do not go along together.)

Besides, the ergo-building of any significant structure, e.g. by a child learning a language, a mathematician digesting a non-trivial idea or a scientist designing a new experiment is a long (mostly unconscious) process, taking weeks, months, years.

This is comparable with the time schedule (weeks) of adaptive immune responses, where "learning" depends on molecular mutation/selection mechanisms. Gerald Edelman [36] proposed a similar principle for learning by the (neuro)brain which, according to Edelman, is based on Darwinian kind of competition/selecton between groups of neurons.

The structure encountered by an invaded organism is a collection of many copies of an essentially *random* object with little correlations between different classes of objects : the profiles of exposed surfaces of antigens. The learning strategy of the immune system – production/mutation/selection of antibodies is well adapted for "understanding" this structure, where "learning" one antigen does not much help to "understand" another one.

But the problems faced and solved by the ergobrain are structurally quite different from selecting antibodies for efficient binding to antigens. The basic function of the brain, consists in finding *correlations/similarities* in flows of signals and factoring away the *redundancies*; this seems hard to achieve by "blind selection" on the realistic time scale. On the other hand, even if this were possible, this would not help us much in unraveling the structures of most ergo-learning processes, such as the language acquisition, for instance.)

A common scenario in the mathematical community is as follows. X gives a lecture, where a (preferably young) Y is present in the audience but who understands nothing of the lecture; every word of the lecture is forgotten next day. (This is what normally happens when you attend a mathematics lecture with new ideas.) A year later, Y writes an article essentially reproducing the subject matter of the lecture with full conviction that he/she has arrived at the idea by himself/herself.

Unsurprisingly, X is unhappy. He/she believes that Y could not arrive at the idea(s) by himself/herself, since Y has no inkling of how the idea came up to him/her, while X is well aware when, why and how he/she started developing the idea. (A similar "structure recall" is common in solving non-mathematical problems, such as "egg riddle" in 3.3, for example.)

The structural patterns of the ergobrain, although being of evolutionary origin, can not be accounted for by the naked survival/selection mechanism, but rather by inevitable constrains on possible ergosystem's architectures; these are, essentially, mathematical constrains.

The structure of the ergobrain is somehow encoded by the neurobrain, but this "code" is by far more complicated (partly, due to its redundancy) than, say, the genetic code. Even if one is granted a detailed description of the physiological brain, (which is unrealistic anyway) along with faithful correlations of neuronal processes with the behavior of an organism one would have no means of learning much about the ergobrain. The problem seems harder than reconstructing the 3-dimensional architecture of a folded protein molecule in space on the basis of a genome sequence, *without* any idea of the 3-space and/or of the interaction rules between molecules.

However, if we assume that the evolution is "big rather than structurally smart" we conclude that the the *"logic"* underlying/generating ergobrain (and ergosystems, in general) can not be too complicated and can be guessed on the basis of ergo-moods (surprised, amused, bored, etc.) that ergobrain induces in our conscious mind.

These moods, being *independent* of the pragmatic content of the signals received by the ergobrain, serve as *universal* signatures/observable of ergo-states. They reflect one's "emotional" reactions to structural peculiarities in the flow of incoming signals, e.g. feelings of being amused, surprised, doubtful, etc.

Although these moods/signatures are "visible" only at the "intake" of external signals by the ergobrain, we conjecture that similar signatures mark and guide the internal ergoprocesses as well.

Besides signatures of ergo-moods, the functioning of the ergobrain is seen in:

(non-pragmatic aspects of) play in animals, children and adult humans,

development of spacial perception in humans and animals,

learning/creating native languages and learning foreign languages,

assimilation/creation of structure patterns by students of mathematics and natural science,

human interaction with art – poetry, music, dance, painting, modulo sex-associated and other ego-ingredients.

Despite the overall structural complexity of the ergo-brain, its modeling may be easier than, for example, deriving a protein structure from its function: proteins live in the molecular world and can navigate there without "understanding" molecular dynamics and quantum chemistry, while we need to model this world with only approximate knowledge of the local interactions between molecules and with limited power of our computers. But, apparently, the electrophysiology of the brain, *when it is reduced to "ergo"*, has no comparable advantage over our mathematical/computational resources.)

Imagine an ergobrain as a builder of a tiling, an "informational jigsaw puzzle", where the incoming signals make potential candidates for inputeces/patterns suitable for building the fuller structure of the tiling (the full structure, if such exists, is unknown to the ergobrain).

A "new *interesting* fragment" is a piece of information that the ergobrain did not have before and/or could not predict coming. On the other hand such piece must meet some structural requirement/expectation by the ergobrain in order to fit into the already built part of the tiling.

If either all incoming pieces of information were (almost) fully predicted by the ergobrain or if it can not predict (almost) anything then it can not continue the building process. The ergobrain becomes *bored* as opposed to *interested.* and/or *amused.* (Boredom is not a trifle matter, it cripples captive animals in zoos and laboratories, psychologically and physiologically [88].)

For example, the the constant sequences

● ●

□ □

are quite boring (where the sequence of square's is slightly less so as it has more features to learn in order to predict it.

A random sequence, e.g.

□ ● ● □ ● ● □ ● □ □ □ ● ● □ □ ● ● ● □ ● ● ● □ □ ● ● □ □ □ □ ● ● ● □ ● □ □ ● ● □ ● □ ●□

is also boring, although a trifle less so than the constant one: you can not learn anything from it, the past experience does not help you to predict the future (but you can play by making some patterns out of it as we make constellations from randomness of stars in the sky). Both sequences are *too* symmetric to be informative where the latter has full *stochastic*, rather than deterministic, symmetry.

(The complete absence of "internal" structure – the full unrestricted symmetry – is a very strong structure in its own right. Groups of permutations are not boring at all. For example, the beautiful mathematics of coin tossing is founded on randomized permutation groups.

"Complete absence" of a structure may point toward yet another structure not only in mathematics. For instance, the evolutionary biologist's motto – "selective survival of descendants of *random* mutants" – tells you, in truth, that there is *no* any specific "mechanism of evolution" except for *quasi*-perfect inheritance of genetic material in reproduction. Amazingly, this "random", let it be only constrained conditional randomness, is what gives the mutation/selection point of view its structural cutting edge which is lacking from the alternative teleological "theories".)

A kaleidoscopic/ornamental repetition of random patterns attracts the human (animal?) eye, even if it is as simple as below.

★●●⊙●□▲□●⊙●●★●●⊙●□▲□●⊙●●★●●⊙●□▲□●⊙●●★●●⊙●□▲□●⊙●●★

An essential ingredient of ergo-learning strategy is a search for symmetry – repetitive patterns – in flows of signals. Even more significantly, an ergo system creates/identifies such patterns by reducing/compressing "information" and by structuralizing "redundancies" in these flows.

An *amazing* and the most interesting event in the sequence

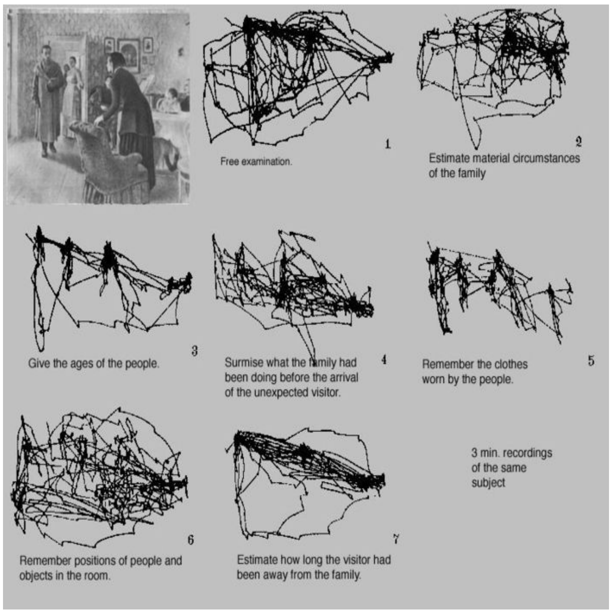
● ●□ □ □ □ □ □ □ □ □ □ □ □ □ □ □ □ □

is the unexpected switch from • to □: your eye stays disproportionally long time focused at this point. Also, your pay most attention to the ends of words and it usually doesn't mcuh mttaer in waht oredr the ltteers in a wrod are. Similarly, your eye spends more time focused at the *edges* of images and bugs love crawling along the edges of leaves.

"Eye movements reflect the human thought processes; so the observer's thought may be followed to some extent from records of eye movements. ...The observer's attention is frequently drawn to elements which do not give important information but which, in his opinion, may do so. Often an observer will focus his attention on elements that are unusual in the particular circumstances, unfamiliar, incomprehensible, and so on."'(Yarbus, taken from Eye Tracking in Wikipedia, also see [50])

Thus various degrees of "interest/amusement" and " surprise/amazement" are rough indicators of a relation of the structure of your egobrain to the informational structure presented in the flow of signals. (If you are bored by a sequence of letters, this may be because you are not familiar with the language or, on the contrary, if you were obliged to memorize this sequence as a child at your school lessons.)

Several months old babies start playing with: "PA PA BA BA". Their



1
Free examination.
2
Estimate material circumstances of the family
3
Give the ages of the people.
4
Surmise what the family had been doing before the arrival of the unexpected visitor.
5
Remember the clothes worn by the people.
6
Remember positions of people and objects in the room.
7
Estimate how long the visitor had been away from the family.
3 min. recordings of the same subject

auditory system records these "PA PA BA BA" with an *amazing* correlation between the sounds and the somatosensory and tactual perceptions. Deaf children start "PA PA BA BA" at the same age but stop doing it sooner: there is less to be amazed with. Eventually, babies get bored with this and start producing more interesting/meaningful sequences of sounds, unless taught that BLAH-BLAH-BLAH is an acceptable adult speech.

Similarly, it is more interesting to run than to walk, to walk than to stand and to stand than to lie: the structure of brain's strategies for keeping your from falling down is the most intricate when you run and the correlation between the visual and somatosensory perceptions is most amazing for the brain in this case.

Our main assumption, in agreement with [54], is that an ergobrain comes to "understand" the world by "trying to maximize" its "predictive power" but what it exactly predicts at every stage depends on what structure has been already built. (If predictability is understood broadly enough, on all levels of underlying structure, then there is no discordance with Rene Thom's "Prédire nest pas Expliquer".)

In order to maximize anything, one needs some freedom of choice, e.g. your eye needs a possibility to run along lines/pages or, in a chess game, you can choose from a certain repertoire of moves. When this repertoire becomes constrained, the ergobrain feels frustrated. This is not visible from the outside, (contrary, say, to the feeling of surprise which is visible on one's face): one can only rely on introspection.

Many of us have experienced an uncomfortable feeling when a lecturer shows slides line by line, preventing you from seeing the whole page. (Such lectures are meant for the heads of Turing machines, rather than for those of humans.) One undergoes a similar feeling of frustration when studying a book or an article (often, alas, written by a mathematician) where the author is purposefully putting a horse blinder on reader's mind's eye. And the poetry by deafblind people tell us how much one should value one's freedom to learn.

("Freedom" for an ergobrain does not mean just a possibility to generate any kind of signals it "wants", but rather to have "interesting" environmental responses to these signals. Thus, for example, a bug crawling on a homogeneous surface of an imaginary infinite leaf has zero freedom: no matter where it goes it learns nothing new. But the presence of a structural feature, e.g. of an accessible edge of the leaf, significantly adds to bug's freedom.)

The "amusing, "amazing","bored"and other ergo-mood signatures seem little to start with, but we shall see that not so little can be reconstructed from these, if one studies the ergobrain by "ergomeans". (Ergobrain's out-coming "Eureka"s, the neurophisilogy of which has been studied [59], are not specific enough to be structurally informative On the other hand, some short time scale perception phenomena, e.g. optical illusions, may carry a non-trivial ergo-message.)

This is not so easy as it looks since it is hard to tell which concepts and ideas are of ergo- and which are of of ego-origin. For example our egomind believes that mathematics is something "abstract and difficult" while the objects we see in front of our eyes are "concrete and simple". However, the ideas of these objects are created by a complicated process of the image building by your visual ergosystem, the result of which is something abstract and artificial from the point of view of your ergobrain, while mathematics is very similar to what ergobrain was doing all its life.

The input of the visual system amounts, roughly, to the set of samples of a distribution of something like a probability measure on the set of subsets of the light receptors in your retina. This "set of samples" is at least as "abstract" as the invariance of the Euclidean 3-space geometry under the orthogonal group $O(3)$, while the reconstruction *set-of-samples* $\rightsquigarrow$ $O(3)$-*invariance* by your ergo-brain is a mathematical endeavor.

(The mathematics of building/identifying the $O(3)$-symmetry of the visual perception field is similar to but more complicated than how Alfred Sturtevant reconstructed in 1913 *linearity of the gene arrangements* on the basis of distributions of phenotype linkages long before the advent of the molecular biology and discovery of DNA [46].)

The difficulty of "abstract" mathematics is, apparently, due to the protective wall separating ego from ergo

Are we Smarter than Apes? Imagine you are subject to a psychology study by an alien who brought you from earth, kept without food for 12 h and put naked into a lighted cubical $3m \times 3m \times 3m$ with perfectly smooth floor, walls and ceiling. There is a cylindrical 1m long stick 1cm thick in the room and a banana attached to the ceiling. Also there is a string hanging from banana with a loop at the hight 2m.

What shall you do to demonstrate your "sapience", knowing that if you fail you will be treated no better than how we human treat animals. (See the answer at the end of 3.2.)

Bird Puzzle: how did Von Neumann do it? A bird flies back and forth between two trains traveling toward each other at 40 and 60 km/h, respectively. The initial distance between the trains is 100 km and the bird flies 100 km/h. What is the distance covered by the bird before the trains meet?

According to an apocryphal story, it took one second for John Von Neumann to come up with the correct answer by summing up the infinite geometric series of distances for the consecutive $\overrightarrow{forth}$ and $\overleftarrow{back}$,

$$D_{bird} = \sum_i (\overrightarrow{d}_i + \overleftarrow{d}_i), i = 1, 2, ..., \text{ where}$$

$$\overrightarrow{d}_1 = 100 \times \frac{100}{160}, \ \overleftarrow{d}_1 = \left(100 - 40 \times \frac{100}{160} - 60 \times \frac{100}{160}\right) \times \frac{100}{140}$$

and

$$\frac{\overrightarrow{d}_i + \overleftarrow{d}_i}{\overrightarrow{d}_{i+1} + \overleftarrow{d}_{i+1}} = \frac{100 - \overrightarrow{d}_1 - \overleftarrow{d}_1}{100}.$$

What a mess!

According to another version of this story Von Neumann found the "smart solution":

The bird spent the same amount of time traveling as the trains, which is $\frac{100km}{(40+60)km/h} = 1h$; hence, it covered $D_{bird} = 100km$.

Then he was annoyed at somebody's mentioning that there was a faster solution.

In fact, there are faster (ergo)solutions, where, moreover, you do not always have to understand all the words in the puzzle.

1. Imagine, your English is poor and you missed all words except for the numbers: 40, 60, 100, 100. Which number would you give in response? Obviously, the best bet is 100, even if you miss the third hundred=40+60.

2. There was only one *distance*-number in the question – 100 km; therefore this is likely to be the *distance*-solution. (This remainins correct even if the distance was 150 km.)

These 1 and 2 are how a baby/animal ergobrain would do it; you need ≈0.3 sec. in either case. And it is not as silly as it may seem to a mathematician: if 100 km stands for a whiff of a predator, you have no time for computing the total length of its expected jumps.

3. An ergobrain equipped with an advanced visual subsystem, (e.g. that of Von Neumann), would mentally unfold the zigzags of the birds' flight, would match this unfolding with the joint motions of the trains, would record that the two are equal and would know the answer immediately after (or even before) being asked: "What is the distance..." , since there is only one distance to ask about in the picture; thus, the problem would be solved in negative time, about −0.5 sec.

4. A physicist would approach the problem by looking for symmetries. Thus, he/she would switch to the (Galilean) coordinate system moving along with one of the trains. Here the second trains moves at 100=(40+60) km/h, while the first one stays still. The bird starting from the moving train at 100 km/h will stay on the train all along and will cover the same distance as the train. (A mathematician would see an error in this reasoning, but the physicist would not care.)

Then our physicist would need another 0.5 sec. to figure out the general solution by exploiting the *scale invariance* of the problem: the distance in question is proportional the initial distance between the trains and also to the speed of bird's flight, while being inverse proportional to the sum of the speeds of the trains. The proportionality coefficient C equals 1, since $C\frac{100km\times 100km/h}{(40+60)km/h} = 100km$, where km/h cancels away in agreement with the logic of 2.

The use of symmetry in physics (e.g. of the above *group* of translations and scalings, which, by the way, also has a "secret" rotational symmetry as we shall see in 3.1) is of the same "ergo-nature" as "jumping to conclusions" in 1, where one continues 100, 100 with another 100. The motto of ergosystems is that of a practicing theoretical physicist:

"imaginative non rigorous mathematics".

1.9 What Kind of Mathematics?

The signals entering the ergobrain via vision, hearing and olfaction are "written" on certain physical/chemical backgrounds the structures and symmetries of which have been much studied by mathematicians.

1. The visual signals are carried by the four dimensional *space+time* continuum. Signals break the symmetry (and continuity) of the *space+time* but eventually, the ergobrain reconstructs this symmetry.

2. The auditory signals are carried by the two dimensional $time+frequency$ space. Ergobrain, unlike mathematicians, does not seem to care about the underlying (symplectic) symmetry of this space; it is concerned with the "infor-

mation content" of these signals and with *correlations* and/or *redundancies* in flows of the signals.

3. The "letters of smells" are aromatic molecules entering our noses. The background space here has by far more symmetries than the the above two spaces. (Its symmetry group is something like the automorphism group of a *Lebesgue-Rokhlin measure space.*) This is, probably, the reason of why the olfactory perception depends on so many different receptors and why there is no human olfactory language.

4. There is no apparent uniform (symmetric) spacial background for somatosensory and touch (haptic) perceptions but their information carrier potential is comparable to that of vision and hearing.

5. Linguistic information entering the ergobrain does not much depend on the physical carrier of this information. This suggest a universal class of structures encoding this information; our main objective is describing these structures, which we call *syntactic ergo-structures.*

Such a structure is a *combinatorial* object X, a kind of a *network* made of finitely many, 10^7-10^9, "atomic units – ergo-ideas". This network structure generalizes/refines that of a *graph*, where some patterns are similar to those found in the mathematical theory of *n-categories.*

The combinatorial structure is intertwined with a geometric one; both an individual network X and the totality of these, say $\mathcal{X}$, carry natural distance-like geometries which are derived from combinatorics and, at the same time, are used for constructing combinatorial relations. This is essential for the *learning process* which is modeled by some transformation(s) $\mathcal{L}$ in the space $\mathcal{X}$; the resulting "educated learner" appears as an *attractive fixed point* X (or a class of points) under this transformation(s).

The (ergo)learning process (transformation) $\mathcal{L}$ starts from a space of signals and eventually compresses (folds) the *information* they carry by some *coclustering algorithms* into our X. When $\mathcal{L}$ applies to visual signals it recaptures the underlying (non-Lorentzian) symmetry of *space+time*; in all cases, ergo-learning creates certain *syntactic symmetries* – rudiments of what one finds in the mathematical *theory of groups.* (Our syntactic structures are more "elementary" and more "abstract" than those studied by mathematical linguists, where "elementary", "abstract", "fundamental", "rudimentary" are synonymous concepts from our (ergo) point of view.)

Building and identifying symmetries *within itself* serves as an essential guideline for an ergo-learner. These are created, seen from outside, by a *statistical analysis of signals*, but the ergo-learner itself can not count (unless being taught) and it knows nothing of probability and statistics. Yet *something like* statistics and probability is essential to an ergo-learner who must distinguish "significant/persistent" signals and ignore "accidental" ones.

(Certain rare signals may be more significant than frequent ones, e.g. a randomish sequence of, say, fifteen letters, such as "gifkumfinkotnid" which appears on three different pages of a book may impress you more than "representations" appearing six times or something like oooooooooooooooooooo coming between lines on twenty different pages.)

The above suggests what kind of mathematics is relevant for modeling ergo-learning; we briefly review some mathematical ideas in section 3, which, although *not directly applicable* to ergo-modeling, point toward avenues of thought one may pursue.

The reader may be surprised by our non-mentioning of "axioms for logical deduction", "Gödel theorem", "automata", "Turing machines", as well as of a logical/philospical analysis of such concepts as "thinking", "understanding", etc. Frankly, these concepts appear to me kind of "anti-mathematical", they do not fit into the folds of a geometer's ergobrain. (We shall explain this in 3.12 on a more technical level.) But they become more accessible when seen against a familiar not so logical background; we shall sketch several "real life" scenarios, which makes these concepts understandable for mathematicians and, we hope, to some non-mathematicians as well.

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2 Structures within and next to Mathematics.

Mathematics is all about *amazing structures* clustered around *symmetries*: perfect symmetries, hidden symmetries, supersymmetries, partial symmetries, broken symmetries, generalized symmetries, linearized symmetries, stochastic symmetries. Two thirds of *core mathematics* (and of theoretical physics) would collapse if the symmetry axes had been removed.

But the peripheral branches of mathematics seen by an outsider are mainly made of useful formulas, difficult computations, efficient algorithms, logical axioms, reliable (or unreliable) statistics..., where the symmetry is diluted to the point of invisibility.

Mathematical concepts and ideas grow like flowers from tiny inconspicuous seeds if cultivated in a fertile soil. We shall demonstrate several instances of how this happens in geometry, algebra and combinatorics, where the *process* of growth, rather than the final result of it, is what will direct our ergo-modeling in the following chapters. Start with two illustrative examples.

2.1 Symmetry in Motion.

Paper on the Wall. A sheet S of paper flatly spread over a plane surface W

e.g. on a table or on a wall can move with three degrees of freedom: one can slide paper in two directions and rotate it with one point pinned to the wall. Nothing hidden about it.

Next, bend a sheet at a corner, i.e. at the joint edge E where two adjacent flat walls W_1 and W_2 meet. One can freely move S along the direction of the edge. Also, one can move it normally to E: start with a sheet flatly spread on W_1 and continually slide it to W_2 across the edge E to W_2 keeping one part of S flatly spread on W_1 and the other part on W_2. Nothing surprising , about it.

Now tightly spread S on two walls so that it sharply bends at the corner and pin it to one of the wall at some point. It seems that you can not rotate it keeping it flatly spread on both walls, the corner would not allow it. But, amazingly, the paper will freely rotate as if there were no corner there.

Similarly, you can rotate a sheet of paper spread around a cylinder or a conical surface. Furthermore, this is also possible (and intuitively clear) for a spherical surface, where instead of paper you use a piece of dry paint not sticking to the surface. On the other hand this will not work (with dry paint) on a surface of an ellipsoid.

But why to engage into a pointless activity of sliding pieces of dry paint on a surface? Well, it is not so trivial as it may appear. It took centuries for mathematicians and physicists to fully understand the properties of such motions and, most importantly, to built up a *structural context* where such motions can be described. This context, as we see it to-day (but, possibly, not tomorrow) is the *Riemannian geometry of curved spaces* in mathematics accompanied by the *general relativity theory* in physics.

The lesson we draw from this example is that if something, however simple, amazes your mind eye (or, rather, your "ergo eye“), there must be an invisible deep structure underlying it.

Our next example is more complicated and a non-mathematician should not be upset if he/she will not understand it, just surf on the wave of words.

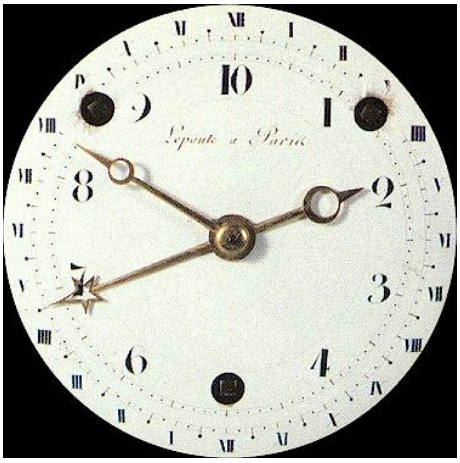
What's Time to a Mathematician? Time, as measured by a calender or a clock, may seem an innocuous mathematical object: time is expressed by a *number* (of years or of minutes) from a point (in the past or in the future) taken for zero. A mathematician, however, is unhappy with the *arbitrariness* of the zero point and of the time unit: why to count from the birth of Newton but not Archimedes? Why to use minutes but not millidays (86,4 sec)?

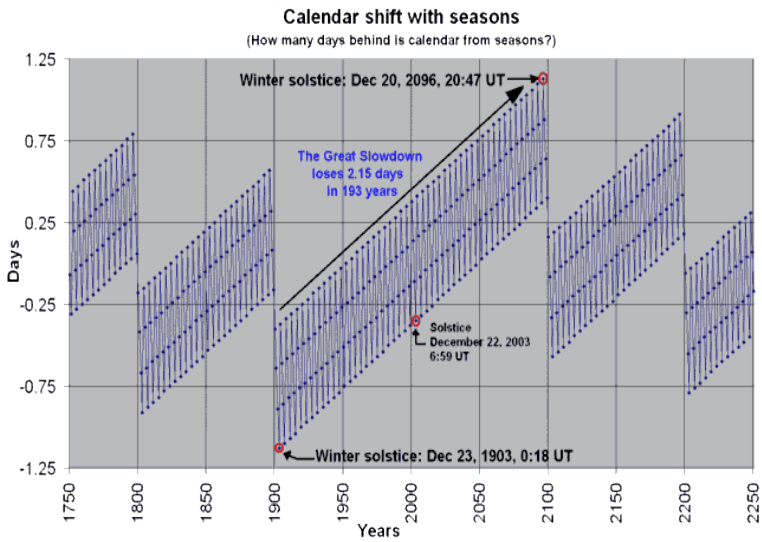
Since there is no preferred zero moment and no preferred unit of time, a mathematician considers *all* possibilities. Every point t_0 on the infinite time line (axes) can be taken for zero in a calender and every *positive* number u_0 may serve as a time unit.

No individual pair of numbers (t_0, u_0) carries a significant structure within itself, but *transformations* of these pairs, as one passes from one calendar to another, (say from the one starting with Newton and measuring time in seconds to the Archimedes calender with years) are more *interesting.*

Namely, let us identify the infinite line representing time with the set of real numbers, mathematicians denote the *real line* by $\mathbb{R}$, and distinguish the two basic classes of transformations applied to the time moments (points) t in $\mathbb{R}$ and to the time units $u > 0$.

(+) *Translations* (or displacements) of the line by certain (positive or negative) amount, say by a, i.e. $t \mapsto t + a$ and u remains unchanged.





These correspond to shifting the origin from one point in time to another by a. For example if we measure time in years and go from Archimedes' to Newton's calender, then $a = -1929 = -(287 + 1642)$.

($\times$) *Rescaling*, i.e. multiplication by a *positive* number b where $t \mapsto bt$ and $u \mapsto bu$.

For example, if we go from minutes to seconds, then $b = 60$.

The full set of these (a, b)-transformations, $(t, u) \mapsto (bt + a, bu)$, can be represented by the b-positive *half-plane*, $\{b > 0\}$, in the coordinate (a, b)-plane. A mathematician call this set, endowed with (the structure of) the *composition rule*

$$(a_1, b_1) \circ (a_2, b_2) = (b_2 a_1 + a_2, b_2 b_1),$$

the (Lie) group of affine transformations of the line. (This is the same group as in a physicists' solution of the bird puzzle in the end of 1.8).

These transformations apply to the half plane represented by the (t,u)-coordinates, where (a,b) applies to (t,u) by the above formula, i.e. (t,u) goes to $(bt+a, bu)$.

Now, this idea goes back to Poincare, one imagines the (t,u)-half-plane (where $u>0$), call it $\mathbf{H}^2$, being an unbounded surface infinitely stretching in all directions (despite the bound $u>0$), something like (but not quite) the ordinary plane, and one regards the above transformations as "rigid motions" (sliding within itself) of $\mathbf{H}^2$.

This is completely counterintuitive and your mind refuses to accept $\mathbf{H}^2$ as a plane-like surface; yet, such point of view is admissible, (only!) because, technically speaking, there is an *invariant Riemannian metric* on $\mathbf{H}^2$. This means that there is a notion of *distance* (also called "metric") in $\mathbf{H}^2$, i.e. an assignment of a positive number taken for distance between every two points, denote it $dist_{\mathbf{H}^2}(\mathbf{h}_1, \mathbf{h}_2)$, which has the familiar properties of the ordinary distance in the plane (e.g. it equals the length of the shortest curve between the points, see 3.6) and such that the (a,b) transformations serve as "rigid motions", i.e. they preserve this distance.

The notion of "Riemannian“ is more subtle and cannot be easily assimilated. Intuitively, it says that such metric, on a (infinitesimally) small scale, is similar (infinitesimally identical) to the distance (metric) in the ordinary (i.e. Euclidean) plane similarly to how the surface of water in a small pond (on *spherical* Earth) appears flat to the eye. (We give a definition of "Riemannian" at the end ??? of 3.6)

If we disregard "Riemannian", we may have many invariant metrics, but there is essentially only one invariant Riemannian metric. (To make it truly unique one has to prescribe distances between three given points in $\mathbf{H}^2$.)

To get the idea of what is going on, forget about t and look at the positive *half line* $\{u>0\}$. This seems, at the first glance, bounded from one side by zero and it can not be moved to the left within itself. However, if we look at the transformation $u \leftrightarrow \frac{1}{u}$ on the u-half-line we see that it "equates“ the apparently finite interval of numbers $0<u<1$ with the infinite one, $1<u<+\infty$.

This transformation, does not preserve the usual distance between points measured by the absolute value $|u_1-u_2|$; however, if we switch from $u>0$ to the variable $l=\log u$ which runs from $-\infty$ to $+\infty$, then the distance measured by $|l_1-l_2|$ is preserved under this transformation, since it corresponds to $l \leftrightarrow -l$ in the l-coordinate.

Moreover, the multiplication by $b>0$ becomes a rigid motion (translation) of the whole (doubly infinite) l-line, $l \mapsto l+\beta$ for $\beta=\log b$, where $dist(l_1,l_2)=|l_1-l_2|$ is preserved under these translations. Thus, $|\log u_1 - \log u_2|$ serves as the distance on the half-line $\{u>0\}$ which is invariant under scaling $u \mapsto bu$.

Coming back to $\mathbf{H}^2$, one can show (this is easy) that the distance $dist_{\mathbf{H}^2}$ on the $t=0$ line in $\mathbf{H}^2$, i.e. between the pairs of points $\mathbf{h}_1=(0,u_1)$ and $\mathbf{h}_2=(0,u_2)$, satisfies

$$dist_{\mathbf{H}^2}((0,u_1),(0,u_2)) = |\log u_1 - \log u_2|$$

for some choice of the base of the logarithm.

All of the above may look heavy, but there is no surprise so far. The *amazing* thing is that

$$\mathbf{H}^2 \textit{ admits rotations} \text{ around every point } \mathbf{h}_0 = (t_0, u_0) \text{ in } \mathbf{H}^2,$$

where such rotations are isometric transformations (rigid motions) which keep a given point $\mathbf{h}_0$ unmoved. (These are similar to rotations of a sheet of paper pinned to a wall or of a layer of dry paint on a spherical surface.)

It is hard, almost impossible, to visualize such rotations. On the other hand, one can write down a formula and check that it does what we want by a simple (and boring like hell) computation. Besides being boring and ugly, such a formula, can hardly be guessed if you do not know what you look for. On the other hand, if you state what you want in general terms, and you properly stretch your planar intuition you can clearly see it with you (ergo)brain eye. (Stretching one's intuition can dislocate the logical joints in the ergobrain; it takes time and patience to learn the yoga of it.)

an (abstract) surface with a Riemannian metric: (not necessarily a part of our space) which admits two independent (continuous families of) rigid motions within itself, can be rotated around each point.

To be honest, there are two exceptions: double-infinite cylindrical surfaces admit no such rotations, only small patches of them do. Also *flat tori* obtained from finite cylinders by gluing together the two components of their circular boundaries have this property.

Thus, our $\mathbf{H}^2$ looks in many respects as the ordinary plane or as a spherical surface $\mathbf{S}^2$ – each of these surfaces admits a 3-*parameter* family of rigid motions (isometries).

(Rotations around all points generate 3-parametric isometries; also they necessarily make the metric *Riemannian* under our standing assumption that the distance equals the length of the shortest curve between ???the??? points).

Furthermore, $\mathbf{H}^2$ (as well as the ordinary plane and the 2-sphere) can be *reflected* with respect to every straight line l in it, where such reflection is an isometric transformation which does not move l and interchange the two halves of $\mathbf{H}^2$ complementary to l.

Mathematicians call this $\mathbf{H}^2$ *non-Euclidean plane* or *hyperbolic plane*, or else the *Lobachevsky plane* the essential properties of which are expressed in terms of the distances between its points as follows.

1. Given two distinct points in $\mathbf{H}^2$, there is a unique copy of the real line $\mathbf{H}^1 = \mathbb{R}$ in $\mathbf{H}^2$ which contains these points and such that the $\mathbf{H}^2$-distance between every two points on this line equals the $\mathbf{H}^1$-distance, where the distance between $\mathbf{h}_1$ and $\mathbf{h}_2$ in $\mathbf{H}^1 = \mathbb{R}$ is the ordinary distance on the line given by the absolute value $|\mathbf{h}_1 - \mathbf{h}_2|$. These $\mathbf{H}^1$ are called *straight lines* in in $\mathbf{H}^2$.

2. Every straight line $\mathbf{H}^1$ divides $\mathbf{H}^2$ into two halves, say $\mathbf{H}^2_+$ and $\mathbf{H}^2_-$ (which depend on the line). Every other straight line $'\mathbf{H}^1 \neq \mathbf{H}^1$ is either contained in one of the halves, or it intersects $\mathbf{H}^1$ at a single point; moreover the intersection of $'\mathbf{H}^1$ with each of the halves $\mathbf{H}^2_+$ and $\mathbf{H}^2_-$ equals a half line in $'\mathbf{H}$.

3. Every translation of a straight line $\mathbf{H}^1$, i.e. the transformation $\mathbf{h} \mapsto \mathbf{h} + \mathbf{h}_0$ for some number $\mathbf{h}_0$ in $\mathbb{R} = \mathbf{H}^1$, extends to a unique rigid motion (isometry) of $\mathbf{H}^2$ which sends each of the two halves $\mathbf{H}^2_+$ and $\mathbf{H}^2_-$ onto itself.

4. The hyperbolic plane $\mathbf{H}^2$ can be *reflected* with respect to every straight line $\mathbf{H}^1$ in it, where a reflection is an isometric transformation which keeps unmoved all points on $\mathbf{H}^1$ and sends $\mathbf{H}^2_+$ onto $\mathbf{H}^2_-$.

These properties (which, accidentally imply "Riemannian") are also satisfied by the ordinary Euclidean plane $\mathbb{R}^2$ (and, with an obvious modification, by the

2-sphere $\mathbf{S}^2$).

The essential geometric difference between $\mathbf{H}^2$ and the ordinary plane $\mathbb{R}^2$ is as follows.

Depart from a point $\bullet$ and go to $\bullet_1$ along the straight segment $\bullet{-}\bullet_1$. When you arrived at $\bullet_1$, turn 90° to the right and go straight the same distance $d = dist(\bullet, \bullet_1)$. Thus, you arrive at $\bullet_2$ within distance d from $\bullet_1$, then repeat the turn and continue going.

If you were in $\mathbb{R}^2$ you would travel along a square $\genfrac{}{}{0pt}{}{\bullet^1}{\bullet}\square\genfrac{}{}{0pt}{}{\bullet^2}{\bullet_3}$ and would arrive at the point $\bullet_4 = \bullet$ of the departure. The last move $\bullet_3 \mapsto \bullet_4 = \bullet$ "cancels" $\bullet_1 \mapsto \bullet_2$ and $\bullet_2 \mapsto \bullet_3$ "cancels" $\bullet \mapsto \bullet_1$. (We are so used to it that we can not imagine how it could be otherwise. But isn't it miraculous that two moves, even if separated by billions light years in space, would "know" how to cancel each other?) But if you do it in $\mathbf{H}^2$ then $\bullet_4 \neq \bullet$.

Geometers were struggling for more than 2000 years since appearance of Euclid's' Elements to show that the parallel postulate (equivalent to $\bullet_4 = \bullet$) *follows* from other axioms. This amounts, in the modern language, to proving that every *infinite* surface ("infinite" is to exclude spheres), which admits a 3-parameter family of rigid motions (equivalently, which satisfies the above 1-4) is isometric to the ordinary plane. This would imply that $\mathbf{H}^2$ could not exist.

Why geometers of the past were missing our $\mathbf{H}^2$? Philosophically, because they trusted into the geometric intuition about the "real world" built into their *(ego)minds*. Technically, because the existence of $\mathbf{H}^2$ is invisible if you start from Euclidean axioms – almost all of *non-trivial* mathematics is axiomatically (logically) unapproachable and invisible.

A more subtle technical reason is that our $\mathbf{H}^2$ needs *real numbers* for the definition of distance, since a distance (metric) on a space X assigns *real numbers* to pairs of points in X.

The first "rigorous proof" of the existence of real numbers, which are needed, for example, in order to define the familiar function $x \mapsto \log x$, such that $x_1 \cdot x_2 \mapsto \log x_1 + \log x_2$, was obtained only in the second half of the 19th century. The most detailed written proof of the existence of $\mathbb{R}$ with all its usual "obvious" properties takes about 100 (boring) pages [63] and, according to [22], one need several more hundred pages to make a truly complete argument.

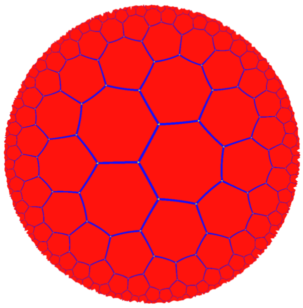
Apparently, "number" is a difficult concept for the human ergobrain, albeit the neurobrain – being a physical model runs on numbers (e.g. electric charges of neurons "are" real numbers). We shall be careful in introducing "numbers" to ergosystems.

Is there anything "visual" corresponding to $\mathbf{H}^2$?

One can find seductive images on the web where a certain piece of $\mathbf{H}^2$ is represented by a surface, called *pseudosphere,* in the ordinary 3-space. These images however, only hamper your understanding: what your visual system conjures out of them brings your farther from the pattern your ergobrain needs to create.

But the picture of tiling of $\mathbf{H}^2$ by equal (in the hyperbolic geometry) *septagons* helps to build up you (ergo)intuition. (The ordinary plane can be tiled by regular hexagons, the humblest of bees knows this, but *not* by septagons.)

The story of $\mathbf{H}^2$ does not tell you much about physicist's time (albeit the isometry group of $\mathbf{H}^2$ equals the $(2+1)$-Lorentz group which operates on the 3-dimensional space-time) but it is instructive in several other respects.



On the negative side, the $\mathbf{H}^2$-story shows you how unreliable our "(ego)mind intuition" is even in such innocuous matters as the space perception where there is no emotional ax to grind: sticking to axioms which look obvious to you is no help in discovering and building new mathematical structures.

Yet, the axiomatic approach is indispensable for making generalizations: no matter how simple and habitual a certain concept or an idea appears to you, it can be *simplified* further by subdividing it into "sub-concepts" and then "forgetting" some of these. (Never mind scarecrow tags of "useless abstraction" brutally nailed to the mathematical offsprings of such logical absent-mindedness.)

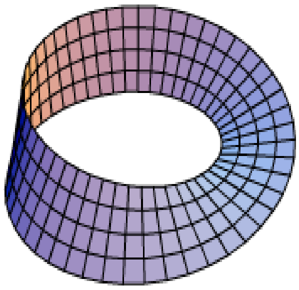
The propensity to overlook that a part of a whole is missing or has changed and, thus, to generalize is not limited to the mathematical reasonong. It is, for example, as much of an asset of the human visual system as the ability to construct Gestalt patterns. If you notice minute details of everything, you may fail to recognize a familiar scene, for instance, if the position of the shadows had changed. (Apparently, this is what often happens to autistic people.)

You never know where a generalization may bring you. But if you are guided by the idea of symmetry, e.g. you trace transformations of apparently boring structureless objects, then you stand a fair chance of arriving at something unexpectedly amazing and beautiful.

2.2 Ambiguity, Galois Symmetry and Category Theory.

The symmetry of the Euclidean 3-space is manifested in every motion of our bodies but most of us know as much of it as the fish knows of water. The mathematical beauty of symmetry emerged not from geometry but from deep waters of algebra.

Abel and Galois discovered, at the beginning of 19th century, (departing from the work by Lagrange and Ruffini) that seemingly non-symmetric algebraic



equations, such as $x^2 - 2x + 3 = 0$ for instance, are intrinsically symmetric, but this symmetry is broken when the underlying *algebraic structure* is symbolically expressed by formulas or equations. The problem with these equations (and formulas in general) is that they are, being quite arbitrary, tell you, if you take them literally, as little about the structures which they express, as the shapes of clocks and/or of calenders on the wall tell you about the nature of time.

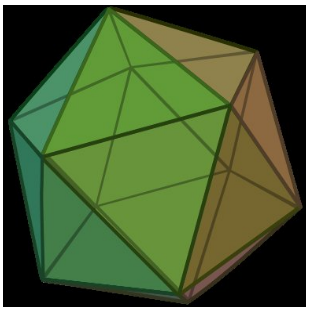
Abel and Galois unraveled the ambiguity inherent into every algebraic equation of degree greater than one – the *ambiguity in choosing one out of several solutions* (roots) of such equation. (There are two roots for the above quadratic equations, $r_{1,2} = 1 \pm 2\sqrt{-2}$.)

Thus, they founded a *mathematical framework* for the study of symmetry, arbitrariness and ambiguity which has eventually grown into the modern *group theory*.

It may seem to a non-mathematician that only Buridan's ass would have any difficulty in choosing one of the two $\bullet$ from $\bullet\ \bullet$. But mathematicians often find this difficult. (See [11] for a mathematician's study of amusing psychological hurdles encountered by mathematics learners and practitioners.) This is not that surprising: try to program a robot doing this, where the two $\bullet$ are not conveniently positioned on a line, you can not just command: "take the left one".

To appreciate the subtlety of the problem, imagine a pair of points continuously moving in 3-space, say $\bullet_t\ \bullet_t$ starting from $t = 0$, such that, at $t = 1$, the pair returns to its original position but the two $\bullet$ switch their places. If, at each moment t, $0 \le t \le 1$, we join the points by a straight segment in space, then the strip swept by these segments makes the (one sided) surface depicted by Listing and Möbiuos 500 years after Buridan. No *deterministic* program (by an easy argument) can choose one of the two $\bullet_t$ (consistently) for all t.

If the Möbiuos strip seems too easy to you, think of how to select one point from a symmetric pair of points $s, -s$ on the surface of the round 2-sphere $\mathbf{S}^2$ (i.e. $s = (s_1, s_2, s_3), -s = (-s_1, -s_2, -s_3)$, $s_1^2 + s_2^2 + s_3^2 = 1$ in the three Euclidean coordinates.) For example, (can you show it?) one *can not* paint $\mathbf{S}^2$ by *three* colors, such that *no* pair of symmetric points $s, -s$ would be *monochromatic*. (Necessarily, there is an s such that both s and $-s$ lie *inside or on the boundary* of a *same* monochromatic domain in the sphere.) This is easy for *two* colors, but the 3-non-coloring theorem was proven (for $(n+1)$-coloring of n-spheres) only in 1933 [12] [67].



The *Galois theory* classifies "algebraic moves" of two or more "abstract • representing roots an equation (e.g. the two complex roots $\bullet_t$ $\bullet_t$ of $x^2+b_tx+c_t=0$ where $b_1=b_0, c_1=c_0$) and organizes equations according to their (groups of) symmetries.

For example the *(absolute) Galois group* (defined at the end of this section) of the equation $x^2-2x+3=0$ has two elements: It consists of all transformation of the two element set – the two roots of this equation. One transformation is the identity map ($r_1 \mapsto r_1, r_2 \mapsto r_2$) and the second one interchanges the roots $r_1 \leftrightarrow r_2$.

The Galois group of $x^3-1=0$ also has two elements. Here, the equation has three distinct roots: ω_1, $\omega_2=\omega_1^2$ and $1=\omega_1^3$, where ω_1 is the first (non-trivial) cubic root of 1, which has, as a complex number, the absolute value one and the argument $2\pi/3$, while ω_2, another cubic root of 1 – the complex conjugate of ω_1, has argument $-2\pi/3$.

The three roots make a regular triangle inscribed into the unit circle in the plane representing complex numbers. The (only) non-identity element in this Galois group is the reflection of the triangle with respect to the line of real numbers, that is ($1 \leftrightarrow 1, \omega_1 \leftrightarrow \omega_2$).

The Galois group of $x^3-2=0$ has 6 elements. It acts on three (distinct) roots of this equations, which are $r_1=\sqrt[3]{2}$, $r_2=\omega_1\sqrt[3]{2}$, $r_3=\omega_2\sqrt[3]{2}$, by all possible transformations – called *permutations*. It is isomorphic the full symmetry (isometry) group of the regular triangle.

Less obviously, the Galois group of the equation $x^5-x-1=0$ has $120 = 1\times2\times3\times4\times5$ elements. It acts by *all* possible permutations on the five distinct roots of this equation.

Amazingly, this is exactly the symmetry group of small *icosahedral* viruses: the full 5-permutation group, is (almost) isomorphic (accidentally?) to the isometry group of the icosahedron, where the following computation makes this plausible.

The icosaherdron has twenty triangular faces and, for every two faces of it, there are exactly six transformations of the whole icosahedron which send $face_1 \mapsto face_2$. This makes $120 = 20\times6$ transformations all-to-gether.

(The actual proof of the isomorphism between the two groups, or, rather, of their normal subgroups of index two, is a trifle more involved and it leads to another interesting story [56].)

Categories, Functors and Meaning. Group theoretic transformations do not reveal much of the structures of the objects we are interested in. For example, there is no apparent symmetry in sentences of a language. However, there are various classes of transformations of sentences and these, up to a certain extent, can be expressed in terms of (mathematical) *categories* (see 3.4).

The concept of a category [70] is a far reaching extension of that of a group, where, in particular, the idea of "something is natural" can be expressed in terms of *functors* between categories. When applied to a language, this leads to a workable (sometimes called "holistic", see p. 242 in [48]) definition of "meaning" of a text without any reference to "real meaning" along the ideas maintained by Zelig Harris: "meanings of words are determined to a large extent by their distributional patterns" [91] [92].

(*Gut feeling* of "real meaning", unlike the "real meaning" itself, which as an ego-concept, does correspond to something structurally significant. For example, when you suddenly perceive an immediate danger, the blood vessels of your colon contract [35]. However, since this was directly observed only in experiments on dogs, human's gut interpretation of "real meaning" of a piece of writing or of a chess party remained unexplored for ethical reasons as much as the gut content of human *qualia*: yet, there is a recorded instance of a mathematician who experiences abdominal pain upon coming across an unrigorous mathematical argument [71].)

Mathematics has its own ego/ergo time-gradient: our theories have been getting closer and closer to our ergo in the course of history.

(Safety, comfort and enlightenment, brought along by the agricultural, industrial, scientific and politically benevolent developments, have undermined the traditional ego dominant mode of human thinking; thus, allowing the luxury of non-utilitarian ergo-reasoning in all domains, not only in mathematics.)

When modeling the ergobrain, we keep in mind this gradient and we shall not shy away from something only because this appears "difficult and abstract" to our egomind. Probably, the full concept of "ergo-brain" can be expressed in a language similar to that of *n-categories* (see ???) which make an even more refined (than categories/functors) tool for dealing with "arbitrariness and ambiguity" .

Understanding mathematics is notoriously difficult for everybody, including mathematicians. It took centuries of concerted efforts of hundreds of human minds to crystallize fundamental mathematical ideas. It takes dozens of pages to properly describe such an idea, e.g. that of a group or of the Galois symmetry. It takes months, if not years, for an individual subconscious mind (ergobrain) to digest and assimilate what is written on these pages.

What is *amazing*, is that this learning time counts in years not in billions of trillions of quadrillions years. No present day model of learning can achieve anything comparable even on the time scale of quintillions of quintillions of quintillions of years. (This gap is similar to and, probably, even greater than that between the efficiencies of industrial catalysts and of natural enzymes.) There is, apparently, a hidden mathematical machine in everybody's brain, an *amazingly simple* machine at that. Our goal is to indicate a possible way such machine may work and our approach will be guided by the *spirit* of the mathematical ideas of *structure and symmetry.*

Algebraic digression for the sake of a curious reader. The Galois group can be associated to an arbitrary n-tuple of complex numbers, $\{r_1, r_2, ...r_n\}$

as a group of certain permutations of these numbers, where, observe the total number of the permutations equals $n! = 1 \times 2 \times ... \times n$. Thus, our group will contain at most $n!$ permutations in it.

The Galois group consists of the permutations which preserve all rational relations between the numbers r_i, $i = 1, 2, ..., n$.

Let us spell it out.

A rational relation between $r_1, r_2, ..., r_n$ is a polynomial $R = R(x_1, ..., x_n)$ in n variables with *rational* numbers for the coefficients, such that $R(r_1, r_2, ..., r_n) = 0$.

For example, $R(x_1, x, x_3) = 10x_1^2 - \frac{3}{7}x_2 + x^8 - 2x_1^2 x_2 x_3^4$ is such a polynomial in the case $n = 3$. The numbers r_1, r_2, r_3 *satisfy* this relation R, if $10r_1^2 + \frac{3}{7}r_2 + r_3^8 - 2r_1^2 r_2 r_3^4 = 0$.

A permutation, say $(r_1 \leftrightarrow r_1,\ r_2 \leftrightarrow r_3)$, *preserves* this relation if also $R(r_1, r_3, r_2) = 0$. Thus, R-preserving permutations in this example are those where either r_1, r_2, r_3 satisfy both equations

$$10r_1^2 + \frac{3}{7}r_2 + r_3^8 - 2r_1^2 r_2 r_3^4 = 0 \text{ and } 10r_1^2 + \frac{3}{7}r_3 + r_2^8 -, 2r_1^2 r_3 r_2^4 = 0,$$

or none of the two.

If the complex (or real) numbers $\{r_1, r_2, ..., r_n\}$ are taken on random, then there is no (non-trivial) rational relations between them (this is easy to show); thus every permutation preserves all relations between them, since there is nobody to preserve. Hence, the Galois group equals the full permutation group with $n!$ elements in it.

At the other extreme, if some of the numbers, say r_1, is rational, than it can not be moved by the Galois group, since such a move, say $r_1 \mapsto r_i \neq r_1$, would break the relation for the *rational* polynomial $x_1 - r_1$.

Galois Group of a Polynomial $P(x) = a_n x^n + a_{n-1}x^{n-1} + ... + a_0$. Here the coefficients a_i are rational numbers and we define this group only if the n roots r_i of $P(x)$ are distinct: the Galois Group of $P(x)$ is declared to be just the Galois group of the n-tuple $\{r_1, ..., r_i, ..., r_n\}$.

The roots of equations with rational coefficients have lots of rational relations. For example the two roots of $x^2 + a_1 x + a_0 = 0$ satisfy

$$r_1 + r_2 + a_1 = 0 \text{ and } r_1 r_2 - a_0 = 0, \text{ since } (x - r_1)(x - r_2) = x^2 + a_1 x + a_0$$

by elementary algebra. Then one can easily see that the permutation $r_1 \leftrightarrow r_2$ preserves all *symmetric* relations, i.e. where R is a *symmetric* polynomial: $R(x_1, x_2) = R(x_2, x_1)$ for all complex numbers x_1, x_2. On the other hand, a rational relations between these roots, $R(r_1, r_2) = 0$, is possible only if R is a symmetric polynomial, provided the roots are irrational. (Too many relations would make the roots rational.)

It follows, that the Galois group of a quadratic equation with distinct *irrational* roots consists of two elements – all permutations of its roots.

In general, the Galois group of a polynomial with rational coefficients equals the full permutation group if and only if all rational relations between the roots are given by *symmetric* polynomials $R(x_1, x_2, ..., x_n)$, i.e. those which are do not change under permutations of the variables. One can say that non-symmetric relations between roots break their symmetry and reduce the Galois group.

(E.g. the polynomials $P_1(x_i) = x_1 + x_2 + ... + x_n$, $P_2(x_i) = x_1x_2 + x_1x_3 + ... + x_1x_n + x_2x_3 + ... + x_{n-1}x_n$, and $P_n(x_i) = x_1 \cdot x_2 \cdot ... \cdot x_n$ are symmetric, as well as combinations of these, e.g.

$$P_1(x_i^3)P_n(x_i) + (P_2(x_i))^2 - P_1(x_i^2)P_2(x_i^4),$$

while $x_1 + 2x_2 + 3x_3 + ... + nx_n$ and $x_1 + x_2^2 + ... + x_n^n$ are instances of non-symmetric polynomials.)

A relevant example of a non-symmetric relation is $r_2 - r_3^2 = 0$. It is satisfied by $r_2 = \omega_1$, $r_3 = \omega_2$ for the above cubic roots of 1 with the arguments $\pm 2\pi/3$, as well as by $r_2 = \omega_2$ and $r_3 = \omega_1$, since $\omega_2 = \omega_1^2$ as well as $\omega_1 = \omega_2^2$. However, this is not satisfied if an ω is replaced by 1. It follows that the Galois group can *not* transform 1 either to ω_1 or ω_2, which reduces the Galois group of the polynomial $x^3 - 1$ to a single (non-identity) permutation $(1 \leftrightarrow 1, \omega_1 \leftrightarrow \omega_2)$.

This example is not very convincing, since 1, being a rational number, is kept fixed by the transformations of the Galois group anyway. A more comprehensive picture is seen for the equations $x^n - 1 = 0$ for $n > 3$. Indeed, let ω be the root with the argument $2\pi/n$ and observe that its powers

$$\{r_1 = \omega, r_2 = \omega^2, r_3 = \omega^3, r_4 = \omega^4, ..., r_0 = r_n = \omega^n = 1\}$$

make the full set of roots. (Geometrically, these are the vertices of the regular n-gon inscribed into the unit circle in the complex plane with 1 being one of the vertices.)

There are lots of non-symmetric relations between these roots, namely

$$r_ir_j - r_k = 0 \text{ whenever } i + j - k \text{ is divisible by } n.$$

It is not hard to show that the Galois group consists of *all* permutations which preserve these relations.

If $n = 4$, the Galois group contains only two elements, since two out four roots, ± 1, are rational and all the Galois group does amounts to interchanging $\sqrt{-1} = r_1 \leftrightarrow r_3 = -\sqrt{-1}$ – nothing new here.

More interestingly, if n is a *prime* number, then the Galois group has $n - 1$ elements in it, since *there are exactly* $n - 1$ *permutations which preserve the* $r_ir_j - r_k$ *relations.*

The latter trivially follows from *Lagrange theorem* (stated at the end of 3.7) the proof of which is easy but *non-trivial.*

("Non-triviality" of an argument is judged, as it is customary in mathematics, by how much it is removed from the mundane ego-staff. Later on, we shall give a more objective measure of "non-triviality" in the general context of ergosystems without any reference to anybody's "ego".)

The definition of the Galois group is transparent but its computation in specific examples may be hard, and it is still unknown which groups of permutations may serve as Galois groups of rational polynomials.

It is not surprising, however, and it is not difficult to show that the Galois group of a "generic" equation with rational coefficients contains *all* permutations of the roots. For example, this group is (almost) icosahedral for generic equations of degree 5.

2.3 Problem with Numbers and other Mathematical Notions.

Mathematical concepts are not perfectly adjusted for expressing non-mathematical ideas. For example, the basic mathematical notions:

"number", "equal", "random", "continuous", "exist" , "all", "infinite"...

crystallize and formalize "intuitive" ideas about the world. But when we "re-crystallize" and apply these notions to the "real world" we needs to keep track of what was lost and what was inadvertently introduced by the "mathematical crystallization" (Similarly, one can not directly apply the results of the X-ray analysis of a crystallized protein to the "same" protein in a living cell.)

For example a number, such as 2, 3 or 4..., has (at least) two different faces to it. It may be the *ordinal number* associated to counting and formalized in the *Peano arithmetic*: the structure of numbers is expressed by the single operation $n \mapsto n + 1$.

But "number" also means *cardinality* of a "finite collection of atomic objects", where the ambient mathematical structure is what is now-a-days called *the category of finite sets*. This structure is richer: finite sets S come along with *maps* f, also called *functions* and/or *morphisms*, between sets, denoted $f : S_1 \to S_2$, where such an f assigns, to each atom s in S_1, an atom, denoted $f(s)$, in S_2.

It is amazing that both languages are "equivalent" but it is not easy to give a *general* definition of equivalence which would be expressible in a computer language and *formally* applicable to the present case. (Even ordering a two element set is a nontrivial task for a Buridan computer.)

On the other hand, this equivalence, when you apply it to the "real world", tells you that no matter how you proceed in counting atoms in a molecule (at a low temperature) or pixels on your computer screen you arrive at the same (ordinal) number. Yet, this "naive" equivalence does not come for free: n objects can be ordered (counted) in $n! = 1\times2\times3\times...\times n$ different ways. If n is moderately small, say $n \leq 10$ you can check this "sameness" experimentally (with a help of a computer for $n = 8, 9, 10$) by counting 10 atoms in all possible ways. But starting from something like $n = 30$ no conceivable experiment can confirm that *all* counts give the same answer. And even when n is small, the representation of a *cardinal* n by the *ordinal* n, introduces the $n!$-ambiguity (arbitrariness) which is, typically, much greater then the information encoded by n itself.

For instance, if you are making a computer program for analyzing black/white images on a (modest) 100×100-pixel screen, and you represent images by, so called, 10^4-"vectors", you add $N = \log_2 10^4!$ "parasitic" bits to $n = 10^4$ bits of information, where $N > 6n$. What is even worse, you burden you program with the task of filtering away additional irrelevant information.

It is unlikely that your brain, while analyzing the image on about 10^8 retina cells, use such "vectors". (In practical programming such ordering is inevitable and is only moderately harmful, since the memories of the modern computers *physically* carry order structures; but there is no reason to expect built-in order structures in the mammalian brains).

Another problems arises when we deal with large sets and large numbers. For instance, you can safely say that there are less than 10^{100} particles in the visible universe but this does not mean that the "number of these particles" is a bone fide cardinal number.

This becomes even more dramatic with numbers $> N = 10^n$, say for $n > 100$, where, notice, that "the set $\mathcal{S}$ of states" of the memory of a laptop computer has cardinality more than 10^n for $n = 10^{10}$. You may say "this $\mathcal{S}$ is finite" but you can not *unrestrictedly* apply the mathematical properties of finite sets to such $\mathcal{S}$, nor can you use your "intuition" of finiteness when arguing about such $\mathcal{S}$, since most states in $\mathcal{S}$ are unreachable in the space/time framework we are confined to. Neither can you operate with $\mathcal{S}$ as if it were an infinite set: the concept of "set" only partially applies to these $\mathcal{S}$.

Listable sets. Summing up, large sets, such as the set of all strings of length 30 in 10 symbols (of cardinality 10^{30}) can not be treated on equal footing with mildly large sets, e.g. the set of 10-symbol long strings: the latter set is *(phisicly) listable*, it can be implemented, kept and manipulated on a computer with > 10 GB memory, while the cardinality of the former (being "only" the cube of the latter) is more than million times greater than the Avogadro number, $N_{\mathrm{Av}} \approx 6 \times 10^{23}$ (roughly, the number of molecules in 18g of water).

Our basic listable example is the set of words in a human language (notwithstanding the fact that agglutinative languages, such as Turkish, count their words in millions), while the sets of possible sentences even of moderate length, say starting from $9 - 11$ words, are hopelessly unlistable.

(As a matter of comparison, the human brain is estimated to contain 10^{10}-10^{11} neurons and, probably, 1000 time more synapses. Thus the people on earth have about N_{Av} synapses all-to-together. If one could use each water molecule as a computer memory cell, one would need more than three tons of water for listing a 10^{30}-set. If this is not enough, take 10^{50}, where the required molecular level memory would be of the size of Earth. And all particles in the known universe will not suffice for containing the set of strings of length 100. See ??? for an analysis of large numbers from a mathematical logician perspective of *ultra-finitism.*)

Besides being unlistable, the "set of sentences of a language" is not a full-fledged set of the mathematical set theory, unlike the set of sheep in a flock, for example. There is no such thing as a "collection" or a "library" of *all* sentences of the English language, even if we limit the number of words in a sentence by something ≈ 10. (One might say that making such library is "in principle" possible but if you accept this "principle" you stop using your reason and your intuition which are not designed to deal with such "principles".)

However, despite all of the above, one can apply mathematics to analysis of large sets, e.g. one productively uses counting/probabilistic arguments in the study of natural languages following ideas of Zelig Harris and contrary to Naum Chomsky's stand [19] that "the notion of a probability of a sentence is an entirely useless one, under any interpretation of this term".

A significant part of the problem according to [20] is that the probability of an individual sentence, even if properly defined, is too small to be regarded as a bone fide number. (Noam Chomsky has identified the fundamental structure underlying natural languages – *context free grammar* – which implements and formalizes the "recursive transformation of sentences" idea which can be traced back to Panini's grammar of Sanskrit (≈400 BCE) and which was embraced and developed by Harris, who was Chomsky's teacher. But Chomsky rejected the statistical approach advocated by Harris.)

This "small probability problem" in linguistic is similar to, albeit is more

subtle than, what happens in the statistical mechanics where the probabilities of individual events are also uncomfortably small, but the counting/probabilistic mechanism works not because some numbers do or do not make sense, but because the overall structure of the object of a study possesses a sufficient structural, let it be broken and/or hidden, symmetry to sustain the probability theory.

For instance, if you toss a coin n times, say for $n = 10^6$, the outcomes of your experiment can be represented by the vertices of the binary n-cube X in the Euclidean n-space. Prior to any probability, you observe that this cube is *fully symmetric*: given two vertices x_1 and x_2, there is a transformation of the cube such that $x_1 \mapsto x_2$, namely the reflection of the cube with respect to the hyperplane (of dimension $n-1$) normally bisecting the straight segment between x_1 and x_2 in the Euclidean n-space.

This leads you to *the conjecture* that the two outcomes have *equal* probabilities. Since the number of outcomes, i.e. the number of the vertices of the cube, equals 2^{10^6}, this makes the probability $p(x)$ of each outcome x equal 2^{-10^6} which is a "physically meaningless" number. But because of the symmetry, the equality $p(x_1) = p(x_2)$, makes sense regardless of the meanings of $p(x_1)$ and $p(x_2)$.

A Real Life Example. A man walking along the country side came upon a shepherd with a flock of sheep. The man greeted the shepherd and said

– You've got a lot of sheep!

Sure – responded the shepherd proudly – nobody could ever count them correctly.

– Come-on, I can – said the man. .

I can not believe this – the shepherd said – but if you count them I will give you one.

The man went on counting for several minutes – you have 125 sheep.

Amazing – said the shepherd – indeed, there are 125 sheep in the flock, choose the animal you like best.

The man brought one to his side.

Now – the shepherd said – will you give it back me if I tell you your profession?

It is impossible, try it– the man replied.

You are a mathematician – said the shepherd.

– How could you guess? – asked the man in surprise.
– First give me back my dog, and then I'll explain it to you.

Questions to the Reader. How small is the probability that the shepherd guessed by pure chance? What would be the likely profession of somebody whose count came up to 126 instead of 125? Would *you* choose a sheep instead of a dog?

(Choosing a distinguished object or a position of an object is an essential operation employed by an ergosystem. Thus, an ergosystem solves the *smarter than ape problem* from 1.8 by positioning the stick vetically in the center of the floor of the cubical. Similarly, our mathematician has proved being ergo-smarter than a wolf, for instance.)

What happens to sets, sheep and numbers turns up in a variety of mathematical contexts. In particular, one must be careful in applying one's "finite counting intuition" to *finite state automata* of the size of the brain. For instance, your computer (as much as your brain) is not *just* a finite automaton, because $10^{10^{10}}$ is nor *quite* a finite number.

(A manifestation of such "not-quite-finitness" would be an "effective" self-mapping on the set of states of such automaton which were *verifiably* injective but not *verifiably* surjective or visa versa. Such selfmappings are, probably, abundant, but no proof seems in sight, where the first difficulty to overcome is to properly define "verifiable".)

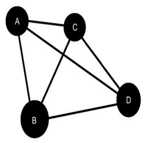
Exponential and super exponential sets. Unlistable sets of strings of listable length and more general arrays of symbols (from a short alphabet) are called *exponential or log-listable sets* . One can *sample* such sets by generating strings in it *on random*; alternatively, one may use some sampling *algorithm* or combine random and algorithmic procedures.

Larger *super exponetial* sets can not be reliably represented by sampling: one hardly can rely on a listable collection of randomly or systematically chosen elements for judging properties of other members of such sets. Yet they naturally appear, e.g. as follows. People may differently answer "Yes or "No" to various questions depending on their personal characters. We distinguish two conceivable human characters if there is a question composed of ten (twelve for safety) words, to which they give different answers. Then the set of all mutually different characters is superexponential. In fact the cardinality of this set equals 2^N where N equals the number of questions. This N is an exponentially large number, at least 10^{30} (with our generous assumption of twelve words) and thus 2^N is superexponential. Even a pure mathematician shudders encountering such huge sets.

I what follows, we shall be very choosy in in our terminology and in assigning basic concepts/operations to our learning systems, where nothing, no elementary structure, no matter how simple looking and "obvious", can be taken for granted.

2.4 What is Equality?

The starting point of finding a structure in a "flow of signals" is identifying "individual signals", such, for example, as phonemes, words and phrases in the



flow of speech, or distinct "objects" in a visual field. Next, one has to decide which signals (e.g. words) are "equal" and which are not. We postpone the discussion of the "signal isolation problem" till ??? and indicate below instances of structures present in "equalities" (compare [70]).

Start with the familiar

$$1+1=2,\quad 2+2=4,\quad 3+3=6,\quad 4+4=8...$$

These look plain and boring to a casual eye, no individuality, no apparent structural beauty.

However, there is a significant structural difference between the first two: $1+1=2$, $2+2=4$ and the other equalities $n+n=2n$ for $n\geq 3$. Namely,

$1+1$ equals 2 in a unique way: the 2-point set has *only* 1 decomposition in two singleton;

$2+2$ equals 4 in three different ways: the 4-point set has 3 decompositions into two 2-point subsets.

In both cases the number d of decompositions is *strictly* less than the number being decomposed, i.e. $d=1<2$ and $d=3<4$.

But starting from $n=3$ the number of decompositions of a $2n$-set into two n-subsets equals $\frac{1}{2}\binom{2n}{n}=\frac{2n\cdot(2n-1)\cdot\ldots\cdot(n+1)}{n\cdot(n-1)\cdot\ldots\cdot 3}>n$ (e.g. $d=10>6$ for $2n=6$). In fact, apart from $1+1=_1 2$ and $2+2=_3 4$, the number d of decompositions of a finite set S of cardinality s into two subsets of given cardinalities satisfies $d\geq s$. (For example, $1+3=_4 4$, $2+3=_{10} 5$, $3+3=_{10} 6$, $2\times 3=_{60} 6$, etc.)

Let us concentrate on $2+2=_3 4$. The "triple of decompositions" yields a "canonical reduction" $4\rightsquigarrow 3$. A classical implication of that, going back to Ferrari, Cardano, Lagrange, Abel and Galois, is the reduction of quartic equations to cubic ones via the homomorphism of the permutation groups, $\Pi(4)\to\Pi(3)$. Below is a fragment of the resulting (Ferrari) formula for a root x of the equation $x^4+bx^3+cx^2+dx+e=0$.

$$\ldots\sqrt{\ldots+\frac{\sqrt[3]{2}(c^2-3bd+12e)}{3\sqrt[3]{2c^3+\ldots-27b^2e-72ce+\sqrt{-4(c^2-3bd+12e)^3+(2c^3+\ldots+27b^2e-72ce)^2}}}}\ldots$$

(No such formula is possible for equations of degrees ≥ 5 by Abel-Galois theory in concordance with the inequality $d\geq s$ for $s\geq 5$.)

A more recent amazing descendent of $2+2=_3 4$ is the world of geometric/topological structures on 4-manifolds (with the Lie algebras splitting relation $so(4)=su(2)\oplus su(2)$ as an intermediate, which says, in effect, that rotations in the 4-space are products of *commuting* 3-dimensional rotations).

The topology of n-manifolds, starting from Milnor's exotic 7-sphere of 1956, has been fully developed by the end of 1960s for $n\geq 5$, where (almost) all geometric problems were reduced to (generously understood) algebra with no (apparent) great mysteries left, with no (known) significant links with other fields and with no(?) futher perspective left open.

It was accepted that the dimensions 2 and 3 were special but it was believed that the high dimensional (surgery and alike) techniques would eventually conquer $n = 4$. (This was confirmed in the topological category [41].)

The landscape has dramatically changed with the appearance of Donaldson's 1983 paper [34]. Not only the dimension four turned out to be quite different and richer in texture than $n \geq 5$, but it grew up full of vibrant connection with a new body of geometry, analysis and mathematical physics. After a quarter of a century, it shows no sign of aging and sprouts new green shoots every 5 - 10 years.

No ergosystem at the present moment would be able to arrive at the Donaldson theory by departing from $2 + 2 =_3 4$, but it is possible that another humble mathematical something, if *properly* represented by an ergosystem, may turn into something unexpectedly interesting.

Egg Riddle. A man comes to a restaurant, orders to eat something slightly bizarre but innocuous – a boiled seagull egg, tries it and soon thereafter kills himself.

What is a possible story behind this macabre scene? Which questions, allowing only yes/no answers, would bring you closer to the solution ?

(We shall tell this at the end of this section where also we will explain what it has to do with the "equality problem".)

In my experience, only children ask questions which leads them straight to the point: a developed egomind biases you toward dozens of irrelevant questions. (Some of my friends solved it with only a half a dozen questions, but I am certain that they heard this puzzle before and forgot about it, since they performed as poorly as everybody else at similar problems. They themselves vehemently denied the fact.)

Let us return to "mathematical" equalities, now in a geometric context.

Given two copies Im_1 and Im_2 of the same image we regard them as equal but the equality $Im_1 = Im_2$ is more elaborately structured object than the above "$=_d$". In fact there are several kinds of equalities in this case.

Assume the two images lie on a flat table T.

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The equality may be achieved by sliding the first image on the table till it merges with the second one. The result of such "slide", can be regarded as a rigid motion – isometry of the infinite (Euclidean) plane extending T; thus, "=" is implemented by an element *iso* of the isometry group $isom(\mathbb{R}^2)$ of the Euclidean plane $\mathbb{R}^2$ and "=" reads "$=_{iso}$".

The group element *iso*, in turn, can be realized by a variety of actual motions of Im_1 to Im_2 within T. Such a motion is a path *pth* in the group $isom(\mathbb{R}^2)$ issuing from the identity element of $isom(\mathbb{R}^2)$ (which keeps the plane $\mathbb{R}^2$ unmoved) to out *is*; this upgrades "$=_{iso}$" to "$=_{pth}$".

However, your eye, while establishing $Im_1 = Im_2$ does none of the above: it rather jumps back and forth between the images "checking " similarity between "corresponding patterns". This is another kind of a structured equality, $Im_1 =_{eye} Im_2$.

Deja Vu.

$$\sqrt{1+2\sqrt{1+3\sqrt{1+4\sqrt{1+5\sqrt{1+\cdots}}}}}.$$

This is the *same* expression as in the first Ramanujan formula from 1.7 and the "equality" between the two is established by an (ergo)mental mechanism which, apparently, involves the memory but the overall (neuro-brain) structure of which remains obscure.

What is perceptible, however, is "the deja vu click" inside your head. Such "clicks" have certain universality but also they may have variable flavors and intensities; also they may carry additional structures, e.g. be accompanied by two distinguishable soft low grunts, if you are a chimpanzee, for instance.

Conversely, if you expect such click and it does not come, you may experience another kind of click – a feeling of being perplexed/amazed upon encountering something new and unusual in a familiar surrounding, e.g. a particular smell, which, probably, can be quantified with something like conditional entropy of the smell signal reaching your brain. (Smell processing by the brain is not a simple matter as it may seem, not even for insects, such as honey bee with the nervous system of $< 10^6$ neurons. Mammals, even olfactory challenged Homo sapience, have tens of millions olfactory receptor neurons in their nasal cavities reaching $\approx 4 \cdot 10^9$ in a bloodhound's nose. [Olfaction, Wikipedia], [3].)

In sum, "equalities/nonequalities" are structural entities which are employed and simultaneously analyzed as well as modified by ergosystems in the course of learning. Our insight into these depends, almost exclusively, on our experience with "the equality structures" in mathematics.

Solution to the "Egg Riddle". The key question is: "Had the man eaten seagull's eggs ever before?" The answer is "no".

– Was he aware that he had never tried them? – No, the expected "deja vu" did not click in his head.

– Did he commit suicide because he thought that he had tried them but just learned he actually hadn't? – Yes, because he realized that what he had once eaten had been something else – *not* seagull's eggs.

A "plausible" filling-in of the rest of the story is a trivial task for a mature egomind.

2.5 Equivalences, Partitions, Graphs, Trees, and Categories.

Symbolically, an *equivalence* is a binary relation which has the same formal properties as equalities,

symmetry: $a \sim b \Rightarrow b \sim a$ (where "$\Rightarrow$" means "implies") and

transitivity: $[a \sim b$ and $b \sim c] \Rightarrow a \sim c$,

except that "equivalence" is defined in the presence of "equality" and it satisfies

$a = b \Rightarrow a \sim b$.

Alternatively, given a set S, e.g. the "set of signals", such as words in a dictionary or retinal images, an equivalence relation on S can be defined via its *quotient map* $q = q_\sim$ which assigns, to each s in S, the *tag* or *label* for $t = q(s)$ that is the *equivalence class of s under "$\sim$"*.

For example, a "tag" attached to a word may indicate the part of speech of the high school grammar: "noun", "verb", "adjective", etc., or it may be a click "animal", "plant", "human",... triggered in your brain by an image on your retina.

In other notations, $q : S \to T =_{def} A/\sim$, where T is the set of tags – *equivalence classes t of "$\sim$"* and where $s_1 \sim s_2 \Leftrightarrow q_\sim(s_1) = q_\sim(s_2)$.

We emphasize the distinction between the binary relation representation and that by the quotient map not for the logical "rigor" sake: the two notions *are* quite different.

The binary relation on a set S of cardinality $|S|$ is represented, literarily speaking, by $|S|^2/2$ bits while a tagging, also called *classification* or *categorization* (in the terminology of Aristotle), i.e. the map, $q : S \to T$, needs only $|S| \log_2 |T|$ bits. (For example, the part-of-speech tagging of a quarter of million words needs only $\approx 10^6$ bits, while the corresponding binary representation requires more than 10^{10} bits.)

On the other hand, an equivalence relation can be represented by a *generator* that is a (simple) graph G on the vertex set A such that

$$a \sim b \Leftrightarrow a \textit{ can be joined with } b \textit{ by a path of edges in } G.$$

Such a G, regarded as a (symmetric subset in $A \times A$) can be comfortably chosen of cardinality $const|A|$ (one can make it with $const = 1$ but this is, usually, unpractical) and thus, represented by $const|A|$ bits. (See [62] for how this, under the name of "categorizations", appears in natural languages and ??? for how an equivalence/similarity is represented by ergosytems in a variety of concrete situations.)

Nested Partitions and Trees. A sequence of equivalence of coarser and coarser equivalence relations on a set S,

$$"\sim_1" \Rightarrow "\sim_2" \Rightarrow "\sim_3" \Rightarrow "\sim_4"...$$

can be represented by a sequence of *onto*, also called *surjective*, maps

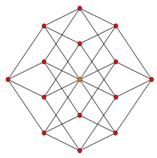
$$S \to T_1 \to T_2 \to T_3 \to T_4...,$$

where the first map $S \to T_1$ is our old quotient map, now denoted $q_1 : S \to T_1$, and the following maps, denoted $q_{i+1} : T_i \to T_{i+1}$ are other *surjective* maps.

Yet another representation of this is given by *(co)nested partitions* P_1, P_2, P_3, P_4... of S, where "partition" means a division of S into "parts" – disjoint subsets of S and where "(co)nested" signifies that the "parts" of each P_{i+1} are *greater* than those of of P_i, i.e. every "part" of P_i is contained in some of the "parts" of P_{i+1}. (This construction can be traced back to Aristotle's classification scheme.)

For example, let $S = \{2^6\} = \{0,1\}^{\{1,2,...,6\}}$ be the set of the 6-digit binary sequences. This S is naturally partitioned into $32 = 2^5$ "parts", with 2 elements each, where each "part" is obtained by "forgetting" the last digit and thus mapping $S = \{2^6\}$ to $T_1 = \{2^5\}$. Then S is partitioned into 2^4 equal parts by dropping the last two digits, where the corresponding quotient map $\{2^6\} \to \{2^4\}$ is the composition of $q_1 : \{2^6\} \to \{2^5\}$ and $q_2 : \{2^5\} \to \{2^4\}$.

This partitioning procedure may be continued until, at the last step, we



arrive at the coarsest possible partition of S with a single "part" which equal S itself and where the map q_6 sends the 2-element set of the numbers $\{0,1\}$ to a one point set denoted $\{\bullet\}$.

Graphs and Trees A collection of sets and maps between them can be represented by a *directed graph* where the vertices are the elements (points) in our sets and where two vertices are joined by a (directed) edge if some of our maps sends one point into the other.

For example a string of maps, such as $S = T_0 \to T_1 \to T_2 \to ... \to T_\bullet$, where $T_\bullet$ consists of a single element, denoted $t_\bullet$, gives rise to a *rooted tree*, where the vertex $t_\bullet$, serves as the root of the tree.

For instance, the above sequence $S = \{2^6\} \to \{2^5\} \to ... \to T_\bullet = \{\bullet\}$ leads to the (standard binary) tree on $127 = 2^7 - 1$ vertices with two incoming edges and a single outcoming edge at all "interior vertices", except for the root $\{\bullet\}$ which has *no outcoming* edges and where the "boundary vertices", called *the leaf vertices* or, just, the *leaves of the tree*, which correspond to the points in S itself, have *no incoming* edges.

If one regards binary sequences as binary representations of integers, then one can "identify" our set $S = \{2^6\} = 2^{\{1,2,...,6\}}$ with the set of 64 numbers, $\{0,1,2,...,63\}$. However, the sets $2^{\{1,2,...,6\}}$ and $\{0,1,2,...,63\}$ are *structurally different* objects: the former, is shaped as a tree which has the symmetry group of $64 = 2^6$ elements corresponding to 6 switches $0 \leftrightarrow 1$, while the latter, makes a string $\bullet\bullet\bullet\bullet\bullet ... \bullet$, which has no (global) symmetries at all (except, if one allows for this, the order reversal transformation $n \leftrightarrow 63 - n$.)

Also, one may think of S as a *binary* 6*-cube*, $\{0,1\}^{\{6\}}$, where $\{6\}$ is an "abstract" (unstructured) set of cardinality 6, but this cube has $2^6 \cdot 6! > 10000$ symmetries.

On Definitions of Graphs and Trees. What makes "definition"? What does it mean that upon seeing/learning one you – your ergobrain – "knows/understand" what is the object being defined?

A good definition must tell you "the truth, all the truth and nothing but the truth", but there is no common agreement, not even among mathematicians, what constitutes a "true definition".

Below are examples of "bad" definitions. (It is easier to criticize somebody's else definition than to come up with a "truly good" one.)

1. (Buffon?) "Man is a bipedal animal without feathers".

This relies on the composition of the *present day* Earth' zoo – it would be the magnificent Tyrannosaurus Rex 65 millions years ago.

The gist of this "definition", of course, is that the two large subsets of the set A of animals: BPA – the set of bipedal animals, which include all birds, and FLA – the set of all featherless animals, have an improbably small intersection,

$BPA \cap FLA = PPL$ – us, people (if we neglect kangaroos). This improbability is fun but it is not especially structurally revealing.

2. "The n-space is *just* the set of n-tuples of numbers".

Even if tacit, "just", is a sign of something being wrong with a definition. (An elephant is not "just" a big mouse with two tails.) Either this is *just* what the author himself/herself understands or what he/she decides the poor stupid reader is suppose to know. If you had no prior exposure to n-spaces, the above definition can make your egomind feel better about your ignorance but it will not add much to the structure of your ergobrain.

More to the substance, the above definition hides "true structures", e.g. symmetries of the space which are apparent in our ordinary 3-space.

3. "Area of a disc is the limit of areas of inscribed regular n-gons for $n \to \infty$".

This "definition" is still lingers in high school geometry textbooks. It is not bad as a *computational device* for finding the area of a disc but it is unsuitable as a *definition*, since an *essential feature* of area is of it making sense for "all" domains, not only for those that are displayed in a particular textbook. Besides, this definition *implicitly* relies on a reader's geometric intuition on the *additivity* of area and its *invariance* under rigid motions. But vocalizing unspoken intuitive assumptions is what a mathematical definition is supposed to do. (Such "vocalization" is not a trivial matter in many cases. It took 22 centuries, counting from Euclid's Elements (≈300 BCE), to mathematically express the idea of area [61].)

One can continue this list indefinitely with

"... is a concept of quantity... ",
"... is the study of the measurement, properties,..."
"... is a variable so related to another..."
"... is an 8-tuple $(\Sigma_V, \Sigma_A, V, ...)$, where..."

It seems, the very idea of making a concise and final definition of something as general as "function", "mathematics", "life", "definition" itself, is deceptive. After all, if something can be explained in a few words it is hardly worth explaining. For this reason, we do not attempt to define "structures" and "ergosystems".

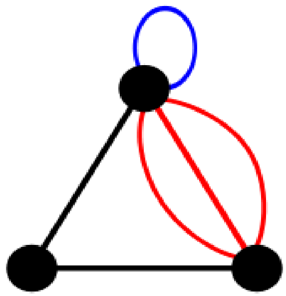
On the positive side, there are structurally well grounded definitions/descriptions, such as

"a *fully recessive allele* which affects the phenotype only when it occurs in homozygous state" in genetics (e.g., p. 4 in [42]),

and

"an *algebraic curve* defined as a *covariant functor* from the *category of algebras* (say, over a given *field*) to the *category of sets*" in algebraic geometry, where this functor must satisfy certain (longish) list of properties which are expressible entirely in terms of the category theory (e.g., [39]).

We shall see later on how *ideas behind mathematical category theory* help to incorporate structural concepts into an ergosystem (such as your ergobrain, for instance).



Now we turn to graphs and fix terminology, where we assume the reader has a visual idea of a graph as

a "collection of objects" called *vertices* or *nodes* where some of them are joined by one or several so called *edges*.

A graph where "several">1 is *not* allowed and where no vertex is joined to itself (by a loop), is called *simple* or (*simplicial*).

The edge itself, pictured as – or as •–•, is a graph with two vertices – the two ends of the edge.

Next come *n-strings*: •–•–• with $n=2$, •–•–•–• with $n=3$, •–•–•–•–• with $n=4$, etc.

The n-gonal graph, $n=1,2,3,...$, is a circle ◯ with n marked points on it for vertices:

1-*gone* or *loop*, i.e. ◯ with a marked point, 2-*gone* ⌓*,* 3-*gon* △*,* 4-*gon* □*, pentagonal graph* ⬠, etc. (The n-gon can be obtained from the n-string by identifying the two end-vertices.)

More representative are Y, with three edges and four vertices, and figure A. The latter inevitably has 4 *(topologically) non-erasable* vertices. where a vertex is called (topologically) *erasable* if it either has a single loop adjacent to it or exactly *two* non-loop edges.

The figure A with the 4 non-erasable vertices makes a *non-simple* graph with 4 edges (where the top $\wedge$ in A is regarded as a single edge). But if the apex of A is taken for a vertex, this becomes a *simple* graph with 5 vertices and 5 edges.

A (simple) *cycle* in a graph is a copy of an n-gone in it. For example, A has a single simple cycle (which is △ if the apex counts for a vertex), while 8 has two 1-gonal cycles; 1-gonal cycles are also called *loops.*

Trees are *connected* graphs *without cycles.* The "connected" assumption may look over-pedantic. Who cares for non-connected graphs!? ("Connected" means that every two vertices can be joined by a string.) But if we exclude disconnected graphs from the definition, the language becomes unexpectedly cumbersome.

A *rooted tree* is a tree with a distinguished vertex regarded as the root of the tree.

Strings, as well as figures Y T, X , + and H are examples of trees where Y and T depict isomorphic trees (graphs); also X and + are isomorphic.

There is a world of difference between graphs and trees: the former are *uni-*

versal in structure – everything can be expressed (albeit artificially in many cases) in terms of graphs. Trees, on the contrary, are structurally *very* special. Only rarely somebody happens to look like a tree, and then one knows a lot about this "somebody". This difference is already seen in the numbers of isomorphism classes of trees and of simple graphs on N vertices for large N: it is, roughly, 3^N for trees (in fact, it is $\sim C \cdot c_0^N \cdot N^{-5/2}$ for $c_0 = 2.955...$ [75]) against $> N^{\alpha N}$, $\alpha > 0.1$, for *simple trivalent* graphs. (Graphs can be obtained by gluing two trees at their leaves: if both trees have M leaves then there are $M! = 1 \cdot 2 \cdot 3 \cdot ... \cdot M > e^{-M} M^M$ possible "gluings" most of which give non-isomorphic graphs.)

Graph structures are ubiquitous in "nature". For example one can make a (simple) graph out of words of a natural language, where the words (in a given dictionary) are joined by an edge if they ever appear as "neighbors", e.g. if one immediately following the other in the texts of a given library. We shall see later on that (an elaborations on) such graph structure contains almost exhaustive information on the grammar and the semantics of the language.

Alternatively, one can consider the graph where there are two kind of vertices: words and, say, the books in a library, where a *word–book* edge signifies that this word appears in the book.

The graphs where there are two kinds of vertices and the edges go only between vertices of different kinds are called *bipartitioned graphs.* If a graph admits a bi-partition of vertices is called *bipartite.*

A much studied graph in the cell biology is where the vertices are (species of) proteins: two are joined by an edge if they interact in some way, e.g. if they "often" binds one to the other.

(Deciding which proteins bind is a non-trivial experimental problem. One method, called *two-hybrid system* runs, roughly, as follows. In order to decide if X binds to Y one uses two auxiliary proteins, say P and Q which are *known* to bind and such that whenever they are bound they perform "something" which is *experimentally observable.* For example $P \vDash Q$ makes the cell to digest a certain nutrient.

Using genetic engineering techniques, X is fused with P and Y with Q, such that (only) binding $X \vDash Y$ can brings P and Q sufficiently close together via $PX \vDash YQ$ (where "only" is achieved by destroying the natural bond between P and Q). If "something" is observed one declares that X and Y bind. In practice, this method is time/work consuming and it is plagued with a multitude of errors which are hard to quantify.)

Only exceptionally, a graph coming from "real life" turns out to be a tree under an examination. If this happens, you hit gold. (This why biologists still fight about the ownership of the phylogenetic tree of life: is it Mendelian – a subgraph in the space of DNA sequences or it is build by Linnaean taxonomy of nested partitions?)

A purely *combinatorial* definition of a graph G, where its edges are stripped of their geometry, is more subtle than it seems. One *can not* define a graph by saying (as combinatorialists often do): "a graph is (just) a pair of sets, V of vertices and E of edges, such that... " (This works, and only up to a point, if there are no loops – eventually problems pop up anyway.)

The subtlety is that an edge, *regardless of its ends* has a non-trivial symmetry which inverts its direction (and which can not be reduced to interchanging the ends of the edge in the case of a loop). A pair of *indistinguishable* points (two directions on an edge) is *not* the same as a single point (undirected edge).

To get an internally consistent definition, think of a graph as being assembled from disjoint edges by identifying some of their vertices. The graph made of disconnected edges is represented by a set E^- (corresponding to the vertices of all edges), where E^- is naturally (tautologically) mapped onto the vertex set V of the graph, say by $\partial^- : E^- \to V$.

Also there is an *involutive* self map $T = T_\leftrightarrow : \circlearrowleft E^-$ (interchanging the ends of the edges) where "involutive" means that $T^2 = id$, i.e. $T(T(\bar{e})) = \bar{e}$ for all $\bar{e}$ in E^-.

With this in mind, a graph is *defined* by

a diagram of maps between two sets E^- *and* V*, written as* $\circlearrowleft E^- \to V$*,*

where the two maps satisfy obvious conditions.

This definition gives the notion of *graph morphisms* between graphs, $G_1 \to G_2$, which agrees with the geometric picture of (edge non-collapsing) maps between garphs: a morphism $G_1 \to G_2$ is a pair of maps, $E_1^- \to E_2^-$ and $V_1 \to V_2$, which are compatible (all diagrams are commutative) with the structure maps defining G_1 and G_2.

We shall give later a definition of graphs (as contravariant functors between categories of sets) which reveals more of their "hidden symmetries"; now we make two remark about tree structures.

Partitions of Leaves. Univalent vertices of a tree are called *leaves* (*terminal nodes* in a rooted tree) where the *valency* (also called *degree*) of a vertex is the number of edges adjacent to it (with each loop counted twice, which is irrelevant for trees as they have no loops.) Every tree X can be canonically represented by a chain of nested partitions of its leaves as follows.

Remove all 2-strings $\bullet\!-\!\bullet\!-\!\bullet$ from X, which have *leaves* at both ends and remove these leaves as well. Call the resulting tree X_{-1}. (If $X = \mathsf{H}$ then the remaining H_{-1} equals the horizontal $\bullet\!-\!\bullet$ in H.) Apply the same procedure to X_{-1}, call the result X_{-2}, and continue with X_{-3}, X_{-3},..., until you arrive at a single vertex or a single edge in X.

The vertex set V_{-i} of each X_{-i} is naturally mapped onto $V_{-(i+1)}$: a vertex v of X_{-i} goes to the *nearest* vertex $v_- = p_i(v)$ in the subtree $X_{-(i+1)}$. (If v remains in $X_{-(i+1)}$, then $p_i(v) = v$; otherwise, $p_i(v)$ equals the vertex in $X_{-(i+1)}$ which is edge-adgacent to v.)

If you retrace your steps and make the tree, say X', out of the maps $V \to V_{-1} \to V_{-2} \to ...$ as we did eralier, then this X' will be not necessarily isomorphic to X; yet the two trees are *topologically isomorphic*: they become (combinatorially) isomorphic if you erase all topologically erasable vertices (i.e. those of valencies 2) in them.

2+2-*Structure on Leaves.* Look closely at the tree H with four leaves. The set $\{4\}_\mathsf{H}$ of the leaves of H is naturally partitioned into two 2-subsets corresponding to the vertical bars (left I and right I) in H.

These "bars", each made of two edges, join the corresponding leaves in H (by the "bars") *without intersecting* each other.

The set $\{4\}_\mathsf{H}$, as we already know, admits three $2+2$ partitions, but the other two do not have the above property: if we join the corresponding pairs of points by paths (3-strings) in the H-tree, these paths necessarily intersect (across the horizontal edge in H).

Now, the nature of a general tree is seen by how it accommodates H-subtrees (along with Y's), i.e. in the structure of the set of (topological) embeddings of H into the tree.

Combinatorially, this looks as a little brother of the $2+2$ *curvature tensor* in the Riemannian geometry:

an assignment of one out of its three $2+2$ *partitions to each* 4*-element subset in the vertex set of a tree.*

Obviously, the $2+2$-structure on the set of leaves L (which is a subset of V) of a tree defines, for every two point subset $\{\bullet_1\bullet_2\}$ in L, a partition of the complement $L\smallsetminus\{\bullet_1\bullet_2\}$ into two subsets, say M and M' in L, such that *every* path from M to M' *meets* the path between $\bullet_1$ and $\bullet_2$. This allows reconstruction of the topology of a tree from the $2+2$ structure on the set of its leaves.

On Categories and Functors. We always understand "categories" as it is common in mathematics, not in the Aristotelian sense. Combinatorially, a category $\mathcal{C}$ is

a *directed graph*, $\overrightarrow{G}$, i.e. "collections" $\{\bullet\}$ of "points" called *objects of* $\mathcal{C}$ and $\{\rightarrow\}$ of directed edges, called *morphisms in* $\mathcal{C}$ between them, e.g. $\circlearrowright\bullet\rightarrow\bullet\leftrightarrows\bullet\rightarrow\bullet\leftarrow\bullet\;\leftleftarrows\;\bullet$, where a morphism m between two objects is often written as $m:\bullet_1\rightarrow\bullet_2$.

A "collection" $\{\triangle\}$ of (disjoint) triangular graphs $\triangle$, where this "collection" is regarded as a (disonnected) graph.

A graph morphism $\{\triangle\}\rightarrow_{\triangle} G$ (i.e. a morphism in the category of graphs defined earlier) where G is the graph obtained from $\overrightarrow{G}$, by "forgetting" the direction of the edges.

This $\rightarrow_{\triangle}$ must satisfy certain properties, where the first one says that

for *every* directed 2-string $\bullet_1\rightarrow\bullet_2\rightarrow\bullet_3$ in $\overrightarrow{G}$, there is a unique triangle $\triangle$ in $\{\triangle\}$ the three vertices of which go onto the three vertices of this string under the vertex map underlying the morphism $\rightarrow_{\triangle}$, where the direction (in $\overrightarrow{G}$) of the third edge coming (to $\overrightarrow{G}$) from $\triangle$ is $\bullet_1\rightarrow\bullet_3$ (rather than $\bullet_3\rightarrow\bullet_1$).

A usual way to say it, entirely in terms of $\overrightarrow{G}$, is that there is a *composition* denoted "$\circ$" between every pair of arrows which make a string:

$\bullet_1\rightarrow\bullet_2$ and $\bullet_2\rightarrow\bullet_3$ *compose* to $\bullet_1\rightarrow\bullet_3$, which is written as $\rightarrow_{13}=\rightarrow_{12}\circ\rightarrow_{23}$.

The second category axiom says that

$$(\rightarrow_{12}\circ\rightarrow_{23})\circ\rightarrow_{34}=\rightarrow_{12}\circ(\rightarrow_{23}\circ\rightarrow_{34}),$$

or

$$(\bullet_1\rightarrow\bullet_2\rightarrow\bullet_3)\rightarrow\bullet_4=\bullet_1\rightarrow(\bullet_2\rightarrow\bullet_3\rightarrow\bullet_4).$$

This non-ambiguisly defines $\bullet_1\rightarrow\bullet_4$ for every string $\bullet_1\rightarrow\bullet_2\rightarrow\bullet_3\rightarrow\bullet_4$.

Finally, one assumes, that every object $\bullet$ comes along with a distinguished *identity morphism* into itself, say ${}^{id}\circlearrowright:\bullet$, the composition with which does not change other morphisms composable with it, i.e. issuing from or terminating at this object $\bullet$.

(This may strike you as a pure pedantry, couldn't one formally introduce such morphisms? The point is that it is not the name but *the position* of this identity in the set of all selfmorphisms plays the structural role in the category, you can not change this position at will.)

The word "collection" in the standard category formalism means "class" in the sense of the set theory, but we shall use it somewhat differently when it comes to (ergo)applications.

A basic notion in the category theory is that of a *commutative diagram.* Firstly, a *diagram* in a category $\mathcal{C}$ is a (usually finite) subgraph D in $\mathcal{C}$, i.e. a set of objects and morphisms between them. A diagram is *commutative* if for every two object $\bullet_1$ and $\bullet_2$ in D and every two chains of composable morphisms, both starting at $\bullet_1$ and terminating at $\bullet_2$, the compositions of the morphisms in the two chains (which are certain morphisms from $\bullet_1$ to $\bullet_2$) are *equal.*

The simplest instance of this is a *triangular* diagram *over* $\bullet_2$ with three vertices and three arrows

$$\overrightarrow{13} : \bullet_1 \to \bullet_3,\ \overrightarrow{32} : \bullet_3 \to \bullet_2 \text{ and } \overrightarrow{12} : \bullet_1 \to \bullet_2,$$

where commutativity amounts to $\overrightarrow{12} = \overrightarrow{13} \circ \overrightarrow{32}$.

Another standard example is a *square diagram* where the commutativity reads $\overrightarrow{14} \circ \overrightarrow{43} = \overrightarrow{12} \circ \overrightarrow{23}$.

A *covariant functor* between two categories $\mathcal{C}$ and $\mathcal{C}'$ is a pair of maps

$$\{\bullet\} \rightsquigarrow_\bullet \{\bullet\}' \text{ and } \{\to\} \rightsquigarrow_\to \{\to\}',$$

which sends the arrows $\bullet_1 \to \bullet_2$ to the corresponding (by $\rightsquigarrow_\bullet$) arrows $\bullet_1' \to \bullet_2'$ such that every commutative diagram in $\mathcal{C}$ goes to a *commutative* diagram in $\mathcal{C}'$.

A *contravariant* functor sends the set of $\bullet_1 \to \bullet_2$-arrows to the corresponding set of $\bullet_2' \to \bullet_1'$ arrows, where the *functoriality* also means preservation of commutativity of all diagrams.

Examples of categories are as follows.

$\mathcal{F}$ – the category of finite sets (one $\bullet$ for every set) and maps (one $\to$ for every map) between them;

$\mathcal{G}$ – the category of groups and homomorphisms;

$\mathcal{S}^1$ – the category of graphs (1-dimensional *semisimplicial complexes*) and the above defined morphisms between them.

Every group Γ defines a category with a single object $\bullet$ and the arrows $\to_\gamma$ for all γ in Γ.

Every category $\mathcal{C}$ and a directed graph D define the category $D^{\mathcal{C}}$ where the objects are D-(sub)diagrams $D_\bullet$ in $\mathcal{C}$ and where the morphisms $D_\bullet \to D_{\bullet'}$ are sets of arrows, say $\overrightarrow{ii'} : \bullet_i \to \bullet_{i'}$ such that the square $\square^{ij}_{i'j'}$- diagrams in $\mathcal{C}$ commute for all ij-arrows in D, i.e.

$$\overrightarrow{ii'} \circ \overrightarrow{i'j'} = \overrightarrow{ij} \circ \overrightarrow{jj'}.$$

An interesting subcategory in $D^{\mathcal{C}}$ is made of *commutative* D-(sub)diagrams in $\mathcal{C}$. In particular, every object o in $\mathcal{C}$ defines the category $\mathcal{C}_o$, where objects are arrows $\bullet \to o$ and morphisms are commutative triangular diagrams over o.

Examples of covariant functors are

(a) $\mathcal{S}^1 \rightsquigarrow_E \mathcal{F}$ which assigns to a graph the set of its edges; and

(b) homomorphisms between two groups regarded as categories.

A basic example of a contravariant functor from any category $\mathcal{C}$ to the category of sets, associated to a given object o in $\mathcal{C}$, is an assignment, to every object $\bullet$ in $\mathcal{C}$ of the set $M = M_o(\bullet)$ of morphisms (arrows) from $\bullet$ to o.

Often, M (functorially) inherits a structure from o, e.g. if o is a group so is M and then the functor lands in the category of groups.

The underlying principle of the category theory/language is that

the internal structural properties of a mathematical object are fully reflected in the combinatorics of the graph (or rather the 2-polyhedron) of morphisms-arrows *around* it.

Amazingly, this language, if properly (often non-obviously) developed, does allow a concise uniform description of mathematical structures in a vast variety of cases. (Some mathematicians believe that no branch of mathematics can claim maturity before it is set in a category theoretic or similar framework and some bitterly resent this idea.)

For instance, a graph G can be defined as a contravariant functor (satisfying some obvious condition), say ϕ, from the category $\mathcal{F}_2$ of sets of cardinality ≤ 2 to the category of all sets as follows. Start with the graph being the disjoint union of the set $V = \phi(\bullet)$ (here $\bullet$ stands for the one element set) where its elements are regarded as vertices and of set $E^{\rightarrow} = \phi(\bullet\bullet)$ of (future) directed edges. Then identify some of the edges and some edge ends according to the arrows in the category $\mathcal{F}_2$ and functoriality.

(This definition allows more morphisms between graphs, namely collapsing edges to vertices which corresponds to $\bullet\bullet \to \bullet$ in $\mathcal{F}_2$.)

What is essential about this new definition is its universality and flexibility. For example, it effortlessly delivers *n-dimensional semisimplicial complexes* for each $n \geq 1$ with $\mathcal{F}_{n+1}$ instead of $\mathcal{F}_2$.

(A representative example of a 2-dimensional semisimlicial complex is the two dimensional counterpart of a loop: the triangle $\blacktriangle$ with its boundary $\vartriangle$ shrunk to a single point. Toplogically, this is a 2-sphere, but it comes with an additional structure the automorphism group of which equals the permutation group $\Pi(3)$.)

Our old definition (with involution T) does not generalize to $n > 1$, essentially because the permutation (automorphism) group $\Pi(n)$ of an n-set for $n > 2$ is *not* commutative. (It is unclear what is the role of *solvability* of $\Pi(3)$ and $\Pi(4)$ in this context.)

Closing the circle, categories themselves can be seen as 2-dimensional semisimplicial complexes:

represent each $\vartriangle$ in $\{\vartriangle\}$ as the boundary of the regular plane triangle $\blacktriangle$ and make the 2-complex by attaching these $\blacktriangle$ to the graph G underlying the category by the map $\{\vartriangle\} \to_{\vartriangle} G$.

(If the category is made of a group Γ, the *fundamental group* of the resulting 2-complex is canonically isomorphic to Γ.)

The category/functor modulated structures can not be directly used by ergosystems, e.g. because the morphisms sets between even moderate objects are usually unlistable.

But the ideas of the category theory show that there are certain (often non-obviuos) rules for generating proper concepts. (You ergobrain would not function if it had followed the motto: "in *my* theory I use whichever definitions I like".) The category theory provides a (rough at this stage) hint on a possible nature of such rules.

2.6 Spaces, Meters and Geometries.

Products and Coordinates. The basic examples of (large) spaces are *Cartesian products* of (small) sets:

given two sets X_1 and X_2, e.g. a three point set $\{\bullet\bullet\bullet\}$ and an eight point one $\{\bullet\bullet\bullet\bullet\bullet\bullet\bullet\bullet\}$, their product $X_1 \times X_2$ is the set of pairs (x_1, x_2) for all x_1 in X_1 and x_2 in X_2.

Thus, schematically,

$$\{\bullet\bullet\bullet\} \times \{\bullet\bullet\bullet\bullet\bullet\bullet\bullet\bullet\} = \{\bullet\bullet\bullet\ \bullet\bullet\bullet\ \bullet\bullet\bullet\ \bullet\bullet\bullet\ \bullet\bullet\bullet\ \bullet\bullet\bullet\ \bullet\bullet\bullet\ \bullet\bullet\bullet\}$$

and

$$\{\bullet\bullet\bullet\bullet\bullet\bullet\bullet\bullet\} \times \{\bullet\bullet\bullet\} = \{\bullet\bullet\bullet\bullet\bullet\bullet\bullet\bullet\ \bullet\bullet\bullet\bullet\bullet\bullet\bullet\bullet\ \bullet\bullet\bullet\bullet\bullet\bullet\bullet\bullet\}.$$

The two products are *not* equal, but they are *canonically isomorphic* via the one-to-one mapping $(x_1, x_2) \leftrightarrow (x_2, x_1)$.

The distinction between "equal" and "canonically isomorphic" may strike you as a silly pedantry. However, if an ergosystem comes across eight triples it does not easily see (if at all) that this is the "same" as three octaves – finding the above "$\leftrightarrow$" needs a non-trivial leap of imagination.

Commutativity of the product is not automatic, but associativity is: all you have to do in order to realize that

$$X_1 \times (X_2 \times X_3) = (X_1 \times X_2) \times X_3 \text{ for } (x_1, (x_2, x_3)) \leftrightarrow ((x_1, (x_2, x_3)$$

is to "forget" the parentheses.

The special case of the Cartesian product is where all "factors" are equal; this is called the *Cartesian power* of a set,

$$X^{\times 2} = X \times X,\ X^{\times 3} = X \times X \times X,\ X^{\times 4} = X \times X \times X \times X, \dots .$$

This, however, suffers from the "mixed structure syndrome" – X is a *set* while the exponents 2, 3, 4,... are *numbers.* A true definition is that of X^S, where *both X and S are sets* and

X^S consists of all maps $S \to X$, i.e. X-valued functions $x(s)$ on S.

If $S = \{1, 2, \dots, n\}$ then there is an "obvious" canonical isomorphism $X^S \leftrightarrow X^{\times n}$. In general, there is an "obvious" (non-canonical) one-to-one correspondence between X^S and $X^{\times |S|}$, where $|\dots|$ denotes the cardinality of a set. But actually exhibiting such a correspondence may constitute a difficult (and unproductive) task.

For example, if X^S stands for (the space of) all conceivable positions $x = x(s)$ of all water molecules s in a bowl X, you can hardly explicitly enumerate the molecules in order to make such a correspondence.

Product and power spaces are used by ergobrains $\mathcal{B}$ for coordinatizations of their perceptions.

Think of an object you hold in your hand, say a small copper bell. The external signal y associated with this bell comes via several channels: visual, auditory, olfactory, tactical.

Imagine four screens in your $\mathcal{B}$, call them X_{vi}, X_{au}, X_{ol}, X_{ta}, with ten pixels in each of them, such that every signal y activates a particular pixel on each screen. Thus the perception of every y is represented by a point $x = x(y)$ in the full perception space X_{vaot},

$$x = (x_{vi}(y), x_{au}(y), x_{ol}(y), x_{ta}(y)) \text{ in } X_{vaot} = X_{vi} \times X_{au} \times X_{ol} \times X_{ta}.$$

Besides, you have a set S of fifty buttons in $\mathcal{B}$ representing your actions: you may shake the bell, bring it to closer to your nose, touch it with a particular finger, etc.

Every such action modifies the perception signal, now you have $|S|$ of them, which gives you a point in the space

$$X^S_{vaot} = (X_{vi} \times X_{au}, \times X_{ol} \times X_{ta})^S = X^S_{vi} \times X^S_{au} \times X^S_{ol} \times X^S_{ta}.$$

This space is huge, it has cardinality $(10^{50})^4 = 10^{200}$ which is by far more than the number of particles in the Universe, but your y is comfortably represented by fifty quadruples of light dots, each on a 10-pixel screen.

Notice, that we substituted S by 50 for the counting purpose, but your $\mathcal{B}$ has no idea how many action buttons were involved and there is no natural order on them. By no means X^S_{vaot} is "just" X^{50}_{vaot}.

Furthermore, the set S comes with its own coordinates corresponding to the muscles being involved in your actions. Given, say, hundred muscles m which can be in two states relaxed/contacted each, your s becomes a point in the space $\{0,1\}^{\times M}$, where M denotes the set of the muscles; accordingly, y goes to the gigantic space

$$\mathcal{X} = X_{vi}^{\{0,1\}^M} \times X_{au}^{\{0,1\}^M} \times X_{ol}^{\{0,1\}^M} \times X_{ta}^{\{0,1\}^M}$$

of cardinality $|\mathcal{X}| \approx 10^{4\times 10^{30}}$.

Moreover, the selection of each consecutive action s by $\mathcal{B}$ depends on what happened on the previous step. This makes s a function on the set $\mathcal{X}$ and you face the triple power space $S^{\mathcal{X}}$ of unimaginable cardinality

$$|S^{\mathcal{X}}| \approx 50^{10^{4\times 10^{30}}}.$$

(X-spaces also come with coordinates but the effect of this is less dramatic.)

Distances and Metric Spaces. Instead of "explicit" constructions of particular spaces one may proceed by introduceing *classes of spaces* according to what one *expects* of the members of such classes and depending on the *language* for speaking about the spaces.

Our present example is that of a *metric space* where one may talk about *points* in such a space and *distances between points*, while the emphasis of the category theory is on maps from one space to another.

The concept of the distance between, say, two locations on Earth looks simple enough, you do not think you need a mathematician to tell you what distance is. However, if you try to explain what you think you understand so well to a computer program you will be stuck at every step.

Everything starts with *numbers*, since a distance is a certain (positive) number and numbers are by no means something simple.

Then, in order to assign a number to a pair of points, e.g. to a pair of locations on Earth, you need to fix a "meter" – a unit of measurement and/or to specify a protocol for measuring distances.

We do not know if there is a canonical "distance meter" in the physical Universe, but our hands gave us the base for the decimal number system, while our feet, arms, thumbs as well as the "wast" of Earth were taken for referential distances – the dominant mammalian species on the third planet of the Sun system has been the measure of all things since Protagoras. (Yet, the Euclidean foundation of geometry was not founded on these dominant mammalian meters.)

The essential property of distance, *the triangle inequality* does not depend, however, of any unit.

$|\overline{*\bigstar}| + |\overline{\bigstar\bullet}| \geq |\overline{*\bullet}|$, where $|\overline{\bullet\bullet'}|$ stand for distances between points.

Motivated by this, one *defines* a *metric space* as a set X with a distinguished *positive* function in two variables, say $|\overline{x_1x_2}| = |\overline{x_1x_2}|_X$, which satisfies this inequality for all triples of points in X,

$$(\triangle_{\geq}) \qquad |\overline{x_1x_2}|_X + |\overline{x_2x_3}|_X \geq |\overline{x_1x_3}|_X,$$

and which is usually assumed *symmetric*, $|\overline{x_1x_2}| = |\overline{x_2x_1}|$. (The latter is not necessarily satisfied in all life examples, e.g. if the distance is measured by the fastest traveling time from x_1 to x_2.)

Also one requires that $|\overline{xx}| = 0$ for all points x in X and sometimes insists (which is too much in some cases) that, conversely, $|\overline{x_1x_2}| = 0$ implies that $x_1 = x_2$.

Besides, it is convenient to allow $|\overline{x_1x_2}|_X = +\infty$ for some pairs of points for saying that x_2 is unreachable from x_1.

The basic example of a metric space is the line $\mathbb{R}$ of real numbers r where one takes the absolute value $|r_1 - r_2|$ for the distance. Notice that the triangle *inequality* for points $r_1 \leq r_2 \leq r_3$ becomes an *equality* in this case,

$$(\triangle_{=}) \qquad |r_1 - r_2| + |r_2 - r_3| = |r_1 - r_3|.$$

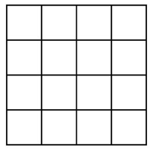
This looks a rather degenerate and boring space, it seems unlikely one could have ever divined the general definition from this example. Amazingly, however, this humble $\mathbb{R}$ begets the *class* of "all" metric spaces in the embrace of the category theory.

2.7 Continuity, Robustness and Fixed Points.

2.8 Similarity, Clustering and Compression of Information.

Similarity is a kind of an "approximate equivalence relation" where the implication $[a \sim b$ and $b \sim c] \Rightarrow a \sim c$ does not always hold. We shall model it below by a simple (finite) graph G where the presence of an edge between two vertices indicates similarity between these vertices.

A cluster W – an approximation to the the concept of an equivalence class – is a subset in the vertex set V of G, such that "most" edges issuing from the vertices in W have their second vertices back in W.



An "ideal" cluster is a *connected component* C of G, where there is no single edge going from C to its complement, and where there are "many" paths of edges between vertices within C.

If a graph comes from "real life" with some similarities being accidental or recorded erroneously (e.g, where edges symbolize similarities between words in a dictionary), one looks for a systematic way of removing "erroneous edges", such that the remaining graph decomposes into connected components, where this decomposition is (more or less) stable under "small modifications" of our criterion for "erroneous".

Some graphs, can not be clusterized: *every* vertex (sub)set W in such a graph either has, roughly, as many outcoming edges as there are edges within this (sub)set or the complement of W in the full vertex set V of the graph has this property. The simplest example is the *full graph on N vertices*, where every two vertices are joined by an edge. Here, every n-vertex set with $n < N/2$ has $n(n-1)/2 < nN/4$ "internal" edges and $n(N-n) \geq nN/2$ outcoming ones.

Surprisingly, *most* connected graphs, say, with $N_{edg} \geq 2N_{vert}$ are like that. At first glance this looks improbable. But if you start believing it and look, for example, at graphs obtained by randomly attaching M edges to (the pairs of vertices of) an N-string (or to any connected graph with N edges, e.g. to a tree with $N+1$ vertices) you see with little effort that if $M \geq N$, then the unclusterable graphs come up with overwhelming probability for large N. (This was first(?) observed in [57].) Now-a-days, these are called *expander graphs.*

On the other extreme, some graphs my have uncomfortably *many* candidates for clusters. For example, if G equals the square lattice (grid) graph on $N \times N$ vertices in the plane, then every $n \times n$ subsquare (there are $(N-n+1)^2$ of these) with a (moderately) large n (say with $n \approx N/2$) may serve as a cluster, since it has $\approx 2n^2$ internal edges and only $\approx 4n$ outcomong edegs.

Expander graphs G are computationally easy do identify with a use of the linear *diffusion operator* $D = D(\varphi)$ on the linear space $\Phi = V^{\mathbb{R}}$ of real function φ on the vertex set $V$of G, where D is (uniquely) defined (via linearity of D) by how it acts on the *δ-functions*: $\delta_{v_0}(v) = 0$ for $v \neq v_0$ and $\delta_{v_0}(v_0) = 1$, where

$$D(\delta_{v_0}) =_{def} val(v_0)^{-1} \sum_i \delta_{v_i},$$

where the sum is taken over all v_i which are edge-adjacent to v_0 and where $val(v)$ denotes the number of edges issuing from v.

Obviously, the eigenvalues of D are positive and the largest eigenvalue of D equals 1 since $D(\varphi) = f$ for constant functions φ. It is also easy to show (after this have been observed [65]) that

if the second largest eigenvalue of D is "small", (i.e. the diffusion is fast) then the graph is "strongly expanding".

On the contrary, if the (maximal) eigenvalue 1 has multiplicity k, then the graph (obviously) has k connected components. More generally, if the diffusion operator D on a graph G admits k eigenvalues which are close to 1, then one can use the corresponding mutually orthogonal eigenfunctions, say $\varphi_i(v)$, $i = 1, 2, ..., k$, of D, for clusterization of (the vertex set of) G.

For example, (if k is small and, thus, 2^k is not too large) one may partition the vertex set V into the maximal subset V_J such that (some of) the functions $\varphi_i(v)$ have *constant signs* on V_J: every $\varphi_i(v)$ is either positive or negative on all of V_J, with some additional provision at the zeros of $\varphi_i(v)$. (This what every linearly minded mathematician would do first, but ergosystems also exploit additional nonlinear clues which may even lie outside G proper.)

In what follows, clusterization means a partition of a set V into what we call "clusters" on the basis of some similarity relation represented by a graph with V being the vertex set. There are many algorithms/receipies for doing this which we shall discuss later on. Now, we shall look on how similarity relations come about.

Sometimes V appears with a metric (distance function) $dist(v_1, v_2)$ (e.g. if V is a subset in a Euclidean space) and then one makes the graph G_{d_0}, $d_0 > 0$, by joining v_1 and v_2 by an edge if $dist(v_1, v_2) \leq d_0$. If one is lucky, there is a (judicious) choice of the *threshold value* d_0, such that the G_{d_0}-clusters are stable under small variations of d_0.

Notice that every (connected) graph (structure) G on a (vertex) set V can be represented by a metric on V: the distance $dist_G(v_1, v_2)$, called the *graph length metric*, is defined as the minimal l such that v_1 and v_2 can be joined by an l-string (path of length l) in G.

A relevant instance of a metric is where the points (elements) in V are represented by by functions on another set U (which may be equal to V in some cases) with values in some set F (of cardinality ≥ 2). This means, V comes with a map into the space F^U of F-valued functions on U, e.g. for the two point set $F = \{0, 1\}$.

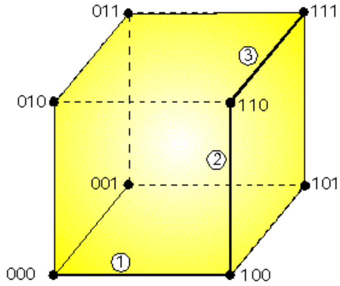
If the cardinalities $|V|$ of V and $|U|$ of U are of the same order of magnitude (or, at least $|U| >> \log |V|$), then the cardinality $|F|^{|U|}$ of F^U is much greater than $|V|$; therefore, the map $V \to F^U$ which represent V by functions on U is likely to be an *embedding*; this makes V a *subset* in the function space F^U.

Spaces of functions often carry various "natural" metrics. The simplest and most general one is the *Hamming metric*, where $dist_{Ham}(f_1(u), f_2(u))$ equals the number of points u in U such that $f_1(u) \neq f_2(u)$. (This metric is informative only for small sets F, with a few elements in them; customary it is defined for finite set U and $F = \{0, 1\}$.)

A metric on F^U, restricted to a subset V of functions, defines, with a suitable threshhold d_0, a graph on V which, in turn can be used to cluster (partition) V.

Let us treat V and U on equal footing. Observe that an F-valued function f in two variables, v from V and u from U, defines a representation of V by F-functions on U, where each v_0 from V goes to $f(v_0, u)$ regarded as function in u. Similarly, U maps to the space V^F of F-functions on V

For example, if (the disjoint union of) U and V serve as the vertex set of bipartite (better to say bipartitioned) graph, this defines a $\{0, 1\}$-valued function



the number of digit positions in which the corresponding digits of two binary numbers or words of the same length are different

$f(u,v)$ which equals 1 if there is an edge $[u,v]$ in the graph and which equals 0 otherwise. Thus one obtains maps $V \to \{0,1\}^U$ and $U \to \{0,1\}^V$.

Similarly, a graph structure on V itself (which is does not have to be bipartite) defines a map from V to the space $\{0,1\}^V$ of $\{0,1\}$ functions on V. Then the Hamming metric on $\{0,1\}^V$ induces a metric on V which is quite different from the graph length metric,

Thus, a graph structure on V may serve for clusterization of V via the Hamming metric induced by the map $V \to \{0,1\}^V$; similarly, a bipartite graph structure on V with U or, more generally, a given function $f(v,u)$, may provide a clusterization of both V and U.

For example, this is common in bioinformatics, V and U may be two groups of (species of) chemical molecules (e.g. proteins) some of which perform in pairs certain "functions" f in cells from a given list F of such functions, e.g. by being involved together in some class f of chemical reactions. Then the value we assign to (v,u) is the function they perform, that is a certain element $f = f(v,u)$ in F.

If the function value set F equals $\{0,1\}$, then, as earlier, $f(v,u)$ is visualized as a (bipartite if $U \neq V$) graph where $[v,u]$-edges correspond to $f(v,u) = 1$ and indicate a presence of a (non-specified) common function of v and u.

Another example, is where v and u are words in a natural language and "1" means that they "often" appear together in a "short phrase".

In what follows, V and/or U will be concrete (listable) sets of moderate cardinalities. To be specific, consider the situation where the cardinalities of U and V are about 10^6 and where we are given a $\{0,1\}$- function, now denoted $v,u \mapsto v \bullet u$ (instead of $f(v,u)$) on, say, 10^9 pairs (v,u) in the product set $V \times U$, such that only 10^8 among them are non-zero. We want to reconstruct/represent $\bullet$, possibly only approximately, on all pairs (v,u) with a use of about $10^8 << 10^{12}$ bits of information. This is, clearly, impossible for general $\bullet$ but, as we shall see later on, many "real life functions" do admit such representations, namely of the form

$v \bullet u = v \star_{ts} u$, where s and t are certain tags (functions) on the sets U and V respectively with the values in some sets S for U and T for V such that the cardinalities of S and T are $\lesssim 10^3$, where $t,s \mapsto f = t \star s$ is a $\{0,1\}$-valued function on $T \times S$ and where $\star_{ts}$ denotes the $\{0,1\}$-function on $V \times U$ induced from $\star$ by t and s.

In other words, we look for partitions/clusterizations of the sets V and U, each into about 10^3 classes/clusters: s for U and t for V, such that the value $v \bullet u$ depends only on the respective classes of (tags on) v and u rather than on v and u themselves.

This is called *co-clustering* of the sets and this obviously generalizes to three and more sets $V, U, W......$, where "*bi-clustering*" is reserved for functions in *two* variables.

Such co-clustering, if possible at all, serves two purposes simultaneously:

1. *Classification.* Co-clustering amounts to partitions of both sets into classes of similar objects, where similarity, say between members v of V is drawn from which u in U they cooperate with.

2. *Compression/Extrapolation of Information.* The 10^{12} bits of information in the above example which are, a priori, needed to encode a function $\bullet$ on all $V \times U$ is reduced to $(2\log_2 10^3)10^6 + 10^3 \times 10^3 \approx 21 \cdot 10^6 \approx 10^8/5$ bits; besides the (co-clustering) construction of this representation (reduction) of $\bullet$ does not use $\bullet$ at all 10^{12} pairs (v,u) but only at 10^8 pairs.

The probability of an $\bullet$, originally defined on 10^9 points, being representable by a $\star$ (with help of s and t) on $10^6 = 10^3 \times 10^3$ points is negligibly small – less than $10^9/\binom{10^{12}}{10^9} < 10^{-10^{30}}$; therefore such representation can not come by chance: it must reflect a certain structure behind a function $f = \bullet$.

On the other hand, it is unrealistic to expect the existence of s, t and $\star$, such that $v \star_{ts} u$ *exactly* equals $v \bullet u$ at *all* 10^9 pairs (v,u) where $\bullet$ was originally defined. But even an approximate representation of $\bullet$ by $\bullet_{ap} = \star_{ts}$ (e.g. with the Hamming distance $dist_{Ham}(\bullet_{ap}, \bullet)$ on all of 10^9-subset of definition of $*$ being small, say $<< 10^8$) is highly significant, since an accidental occurrence of even such approximate representation of $\bullet$ by $\star$ is very unlikely.

(Coclustering is a particular case of "parameter fitting", where one tries to express an experimentally given function $\bullet$ – a set of numbers – as an approximate solution of a system of equation, symbolically $E(\bullet; \star, s, t...) = 0$, where $\star, s, t, ...$ is a set of parameters which must fit the equation. This is justified if the equations E are "simple, natural and non-degenerate" and if the "information content" of $\bullet$ – "the number of parameters" – is significantly *greater* than that of $\star, s, t.....$ This is usually done by *assuming*, better explicitly than tacitly, that $\bullet$ is approximately *constant* along parts of its domain of definition due to the presence of some symmetries within these parts.)

Finding a good approximate reduction of a function $\bullet$ to a $\star$ on small set is, in general, a computationally very difficult problem. However, this can be solved (and it is routinely solved by our ergobrains) in a variety of special cases by exploiting particular features of $\bullet$.

For example, suppose that there exist subsets V_0 in V, and U_0 in U of cardinalities about 10^4, such that their product $V_0 \times U_0$ in $V \times U$ has an overlap of about 10^7 with the set in $V \times U$ where f was defined with about $!0^6$ non-zero values of $\bullet$ in this overlap. Here one may start with biclustering U_0 and V_0 and then proceed with an extension of this partial biclustering to V and U by using, at some point, maps $V \to \{0,1\}^{U_0}$ and $U \to \{0,1\}^{V_0}$ instead of $V \to \{0,1\}^U$ and

$U \to \{0,1\}^V$.

All of the above can rarely be used the way it stands, but various refinements, modifications, generalizations and iterations of co-clustering processes are implemented and employed by learning ergosytems as an essential mean of compression and/or extrapolation of information.

To see why such compression (extrapolation) inevitably must be at work in a sufficiently advanced ergosystem, notice that the volume of the human memory hardly exceeds 10^{12} bits (judging by a possible number of synapses being effectively involved). The total amount of, say linguistic, information we accumulate during the lifetime must be significantly less than that: granted, optimistically, we read/hear a sentence every second this adds up to less than 10^9 in 30 years. Yet, we can (can we?) easily decide whether four given words from a 10^5-word dictionary may consecutively appear in the language. This suggests that our ergobrain employs a fast co-clustering kind of algorithm that reduces 10^{20} units of information to (something much less than) 10^9.

Actually, it *feels* easier to decide linguistic admissibility of longer strings, say, made of 4 - 6 words, but this may be an illusion. There is no apparent convincing experiment: longish "random" strings are, clearly, non-admissible with overwhelming probability. If there are "questionable strings" we do not know how to systematically generate them, but possibly, linguistically gifted people can find sporadic examples of such strings.

It is virtually impossible to make random grammatical sentences at will without actually throwing dice. (Chomsky's "colorless green ideas sleep furiously", where *every* two consecutive [words]/[their antonyms] are strongly correlated, is a witness to this.) In fact, it is *unclear* how many sentences a speaker can normally generate, despite the often made "infinitely many" pronouncement. (My reading of this "infinitely many" is "with manifestly positive entropy".) The very definition of the "number $N(n)$ of sentences in n words one *can* make", say for $n >> 4$, depends on the meaning assigned to "can" where such assignment is a subtle *mathematical* issue (more involved than Kolmogorov's algorithmic definition of probability [85]). Conceivably, $N(n)$ grows *sub*exponentially, i.e. the entropy is (manifestly) zero, for certain "can" and some specific models of speaker's ergobrain.

Even an evaluation of a string is not straightforward, since every sentence, no matter how ridiculous, becomes palatable within an additional (self)explanatory framework. Thus, "colorless green ideas sleep furiously" on 30 000 Google pages, and if you do not expect meeting "an experienced dentist at the wheel" in print anywhere (except for pages of an advanced course in English), "an experienced dentist but a novice at the wheel" is a bona fide cartoon character. (This feature is characteristic of natural languages which have built in self-reflection/self-reference ability. An absurd sequence of moves, for example, can not be included into a true chess party.)

The heuristic/probibilistic reasoning we used above for justifying outcomes of biclusterings and for inevitability of "compression of information" is very convincing but it may be deceptive at times.

To see what may go wrong with counting let us "prove" that the relation $a(b+c) = ab + ac$ is impossible.

Let V be the set of vertices of the regular n-gon in the plane with one point taken for 0 and consider a function in two variable in V which takes values in

V and is denoted $v_1, v_2 \mapsto v_3 = v_1 * v_2$. Then take the $(n-1)$-gon, say W with such a function denoted $w_1 \star w_2$.

Observe that every one-to-one map from W onto V minus 0, say $x : W \leftrightarrow V \smallsetminus 0$, transports the function $\star$ from W to $V \smallsetminus 0$, where it is denoted $\star_x$, and let us evaluate the probability for solvability of the following equations in x,

$$v_1 \star_x (v_2 * v_3) = (v_1 \star_x v_2) * (v_1 \star_x v_3)$$

which we want to hold for all $v_1 \neq 0$ and all v_2 and v_3 in V.

The probability of each one of the above equations, for a single triple v_1, v_2, v_3, to be satisfied equals $1/n$, since the left hand side and the right hand side of the equation take values in an n-element set.

Since the equations must apply to *all triples* with $v_1 \neq 0$, their number equals $(n-1)n^2$, and we might expect that the probability of all of them being satisfied is of order $(1/n)^{(n-1)n^2}$, provided these equations are "roughly independent".

Finally, since $n^{(n-1)n^2}$ is incomparably greater than the number of our maps x which equals $(n-1)! < (n-1)^{n-1} << n^{(n-1)n^2}$, the "inevitable" conclusion is that our equations *admit no solution x for large n.*

Let us apply this to particular $*$ and $\star$. Namely, identify V with the set of integers modulo n, i.e. let $V = \mathbb{Z}/n\mathbb{Z}$ and let $*$ correspond to addition of numbers in $\mathbb{Z}$, where adding m rotates the n-gone by the angle $2\pi m/n$. Similarly, let $W = \mathbb{Z}/(n-1)\mathbb{Z}$ with $\star$ also corresponding to addition of integers.

Now, amazingly, despite our counting argument, the above equations *may* have solutions x: they are solvable whenever n is *a prime* number, because the *multiplication* of non-zero elements in $\mathbb{Z}/n\mathbb{Z}$ makes a *group* out of $\mathbb{Z}/n\mathbb{Z} \smallsetminus 0$, which, moreover, is *isomorphic* to the *additive* group $\mathbb{Z}/(n-1)\mathbb{Z}$ in the case of *prime n* according to an old *theorem by Lagrange.* Thus, an isomorphism $x : W_{[+=\star]} \leftrightarrow V \smallsetminus 0_{[\times = \star_x]}$ satisfies our equations, since $v_3 \times (v_1 + v_2) = v_3 \times v_1 + v_3 \times v_2$.

2.9 Edges, Arrows, Orders, Sums and other Relations.

What is an Arrow. Why do we need two kinds of edges, directed and undirected ones? Is an arrow in a directed graph is an edge with a "direction tag" (whichever this means) or, on the contrary, is an edge just an arrow with "forget it" tag? Or, else, is it a pair of oppositely directed arrows?

Undirected edges in an ergo-architecture usually symbolize *co-occurrence* of events. Making an arrow corresponds to ordering the two events.

The *oder relation* – one of the simplest mathematical concepts (often called *partial order*), which is also present in most (all?) human languages, seems simple enough:

an order, is a binary relation, say "<", such that

$$[a < b \ \& \ b < c] \Rightarrow [a < c].$$

An order is *linear* if $a \neq b$ implies that either $a < b$ or $b < a$.

Shouldn't "order" be among basic ergo-ideas along with "similarity" ?

The catch is that our ability to distinguish "$a < b$" and "$b < a$" depends on spacial order on the letters given beforehand. If you do not have an a priori idea of spacial or temporal order, you can not introduce this concept at all. (Nor can you have the concept of implication "$\Rightarrow$" for this matter.)

Also "orders" are acted upon by the two element group $\mathbb{Z}_2 = \mathbb{Z}/2\mathbb{Z}$ and breaking this symmetry is not so easy as we saw in 3.3.

(Witnesses to this are systematic ”$<$” $\leftrightarrow$ ”$>$” typos in mathematical papers. Similarly, it is hard to distinguish ”left” and ”right”, and, e.g. to remember what is ”above” and what is ”below” in a foreign language, such as ”au-dessus” and ”au-dessous” in French.)

If we ”divide” an order, which is a *binary* relation, by the $\mathbb{Z}_2$-action, we arrive at a *ternary* relation expressing $a < c < b$ by c being *between* a&b which is symmetric for $a \leftrightarrow b$.

Another problem is that an order relation on a moderate listable set V, say, of cardinality 10^8, admits, a priori, no listing itself, since 10^{16} is an unlistable cardinality-number. This may look not very serious as the most important *linear* order can be implemented by a map of V into *numbers.* But ”number” is a very difficult concept for an ergosystem, at least for a human ergobrain. ”Arrowing an edge” is not a trifle matter for an ergosystem.

Arrow in our models of ergosystems, such as $\bullet_1 \to \bullet_2$, often symbolize temporal processes or ”commands”, e.g. to go from $\bullet_1$ to $\bullet_2$. Even if lots of arrows are composable, only a few of these compositions may be performed by an ergosystem at a given point in time; thus the (un)listabilty problem does not arise.

It also may happen that an edge in an ”ergo-graph” joins two distinctly different objects and one is tempted to assign a direction to this edge. However, this may bring confusion.

For example, it is OK to have the arrows **boy→his** along with **girl→her** as well as **go→went** with **have→had**.

However, there is nothing in common between the directionality of the arrows in **boy→his** and **have→had**.

The structure on edges in this example is more subtle than just giving them directions. (It is expressible in terms of a kind of *gauge group* acting on sets of directions, see ???.)

On Numbers, Sums and Functions in Many Variables. Our ergosystems will have no *explicit* knowledge of numbers, except may be for a few small ones, say two, three and four. On the contrary, neurobrains, being physical systems, are run by numbers which is reflected in their models, such as neural networks which sequentially compose *addition of numbers* with functions in *one variable.*

An unrestricted addition is the essential feature of ”physical numbers”, such as mass, energy, entropy, electric charge. For example, if you bring together 10^{30} atoms, then, amazingly, their masses add up (slightly corrected by Einstein's $E = mc^2$) across 30 orders of magnitude; moreover, they ”know” how to add together even if *you* do not to know the exact number of the terms in the sum.

Our ergosytems will lack this ability. Definitely, they would be bored to death if they had to add one number to another 10^{30} times.

But the 10^{30}-addition, you may object, can be implemented by $\log_2 10^{30} \approx 100$ additions with a use of binary bracketing; yet, the latter is a non-trivial structure in its own right that our systems, a priori, do not have. Besides, sequentially performing even 10 additions is boring. (It is unclear how Nature performs ”physical addition” without being bored in the process.)

Disability to add many numbers is a serious shortcoming for several reasons. First of all, adding a non specified number of many small terms ensures, if properly implemented, a robustness of a system. Second of all, addition is the basic (if not the only) systematically present function in many variables. Other functions in the "real world" are compositions of addition and functions in few variables.

(David Hilbert suggested, in his *13th problem*, a version of the question: "Are there natural functions in many variables which admit no meaningful reduction to addition?" It is still remains unclear how to properly *formulate* this problem.)

On "Equivalent" Structures. Speaking of equivalent representations of structures, one often make some tacit assumptions about the meaning of "equivalence". For example, take two introductory books Ch_1 and Ch_2 on chess openings. When can you regard them or their contents as equivalent?

May the two be written in the different languages? May they be using different notational/pictural conventions? May they having different lists of examples? What is a *formal general definition* of "equivalence between Ch-books which would be understandable by a "smart" computer program?

A more innocuous example, we met earlier, is that of a simple graph on a vertex set V. Is it a *symmetric* subset in the Cartesian product $V \times V$, or a subset in the *symmetrized* product, i.e. the set of non-ordered pairs of points in V? Is it a diagram of maps between two sets as in 3.6? Is it a $\{0,1\}$-valued functions in the variables v_1 and v_2?

One can say in this case that the four definitions are equivalent in the category theoretic sense, but then you have to teach your "smart program" some category theory. Besides, in the $\{0,1\}$-function case your program must know what numbers are. (You may object, that 0 and 1 here are not numbers but just members of a two element set. Well, explain this to your program.)

Our point is that ergo-implementations of "equivalent entities" may be quite different; for this reason, we must be very careful in representing any kind structure, no matter how simple it appears, e.g. that of a graph or of a category.

2.10 Spaces of Structures.

"Structured objects" do not appears in isolations but rather as members of "communities" of "similar objects", where the "position" of an object in its community determines the essential (sometimes all) features of this object and where "position" is expressed in terms of the structure(s) of the community.

For example an "individual number", e.g. 1729, does not make much sense; only the set/system of "all numbers of a given type" allows a (structurally) meaningful statements about a particular number. ($1729 = 1^3 + 12^3 = 9^3 + 10^3$ is the smallest number expressible as the sum of two cubes in two different ways as Ramanujan once pointed out to Hardy who could not find anything interesting in this number himself.)

Less obviously – it took a few thousand years for mathematicians to realize this – the whole system of integer numbers $\mathbb{Z} = \{... - 3, -2, -1, 0, 1, 2, 3, ...\}$ also become incomparably more interesting in the community of similar systems – *rings of algebraic integers*, as much as the "ordinary space" acquires new facets of beauty when seen surrounded by other Riemannian spaces.

(Rings of algebraic integers arise from systems of algebraic equations which "reduce multiplication to addition",

$$\times_{ij} \qquad x_i x_j = \sum_{k=1,\ldots,d} N_{ijk} x_k,$$

where N_{ijk}, $i, j.k = 1, 2, ..., d$ are integers $... - 3, -2, -1, 0, 1, 2, 3, ...$

If $d = 1$ this system amounts to the single equation $x_1^2 = Nx_1$ which says, in effect, that x_1 is an ordinary integer.

If $d = 2$ and $x_1 = N$ is an integer, the system $\times_{ij}$ reduces to the quadratic equation $x_2^2 = M + N_2 x_2$ for an integer M divisible by N.

But starting from $d = 3$, this system, even assuming x_1 is an integer, becomes *overdetermined*, since it contains $d(d-1)/2$ equation in $d-1$ variables $x_2, x_3, ...$, where $d(d-1)/2 > d - 1$ for $d \geq 3$; thus, one may expect that these equations have only very degenerate solutions.

Amazingly, for each $d = 2, 3, ...$, there are lots of arrays of integers N_{ijk}, $i, j, k = 1, 2, ..., d$, where the equations $\times_{ij}$ are satisfied for *all* $i, j = 1, ..., d$ by some complex numbers $x_1, x_2, ..., x_d$ which are $\mathbb{Z}$*-linearly independent* in the sense that $M_1 x_1 + M_2 x_2 + ... + M_d x_n \neq 0$ for integer M_i, unless all $M_i = 0$.

If x_i satisfy all $\times_{ij}$ with some N_{ijk}, then the totality of numbers $M_1 x_1 + M_2 x_2 + ... + M_d x_d$ for all d-tuples of integers M_i, is called a *ring of algebraic integers*, provided a certain minor conventional condition is satisfied.

Every such ring can be seen geometrically as the *lattice of points with integral coordinates* in the Euclidean d-space, denoted

$$\mathbb{Z}^d = \mathbb{Z} \oplus \mathbb{Z} \oplus ... \oplus \mathbb{Z} \text{ in } \mathbb{R}^d = \mathbb{R} \oplus \mathbb{R} \oplus ... \oplus \mathbb{R}.$$

The simplest instance of this for $d = 2$ is the lattice of complex integers $M_1 + M_2\sqrt{-1}$, where the domain of all complex numbers, denoted $\mathbb{C}$, is identified with the 2-dimensional Euclidean space, i.e. the ordinary plane,

$$\mathbb{C} = \mathbb{R}^2 = \mathbb{R} \oplus \mathbb{R}.$$

The set of real numbers $M_1 + M_2\sqrt{2}$ also makes our ring, but its geometric representation by a lattice of integer points (M_1, M_2) in the plane $\mathbb{R}^2 = \mathbb{R} \oplus \mathbb{R}$ may seem artificial. In fact, the multiplication of the numbers $M_1 + M_2\sqrt{2}$ does not extend to any "nice and natural" multiplications in $\mathbb{R}^2$ unlike the case of $M_1 + M_2\sqrt{-1}$. However, there is a "$\sqrt{2}$-natural" geometry in $\mathbb{R}^2$ coming from a *Lorentzian pseudo-distance* which makes this representation more palatable.

A simple example for $d = 3$ is provided by the three cubic roots of unity $\omega_1 = 1$, ω_2 and $\omega_3 = \omega_2^2$, where the $\times_{ij}$-equations they satisfy are

$$\omega_2^2 = \omega_3, \ \omega_3^2 = \omega_2 \text{ and } \omega_2\omega_3 = 1$$

and where the corresponding lattice fits into the 3-space $\mathbb{R}^3 = \mathbb{R} \oplus \mathbb{R} \oplus \mathbb{R}$ with a suitable "ω_i-geometry" in this space.)

Examples of structurally organized classes of structured objects can go for ever, but nobody has ever seen an interesting "isolated structure". Apparently, *anything* unique of its kind is a structurally uneventful "anything".

(Being "structurally interesting" has little to do with correctness and/or usefulness of an idea. For instance, the belief in the "reality of the external world" is indispensable for our survival but every discourse about "reality of this reality" soon deteriorates into a structurally unbearable bore. Similarly, polytheistic mythologies may be just fairly tales, but they are more entertaining than the moralistic dogmas of monotheism. And the crystal idea of Platonic truth, ingrained into the self-justification shield of every working mathematician, does not ring the structure bell, but a contorted conglomerate of formal deductions in mathematical logic sometimes does.)

The "communal structure" most prominent in an algebraist's eye is that of a category, where objects come along with sets of morphisms between them and where the detectable properties of the objects depend on what are declared "morphisms".

For example, if your objects are metric spaces, then there are many (too many?) possible choices for morphisms which come up in different branches of mathematics: *isometric maps* are studied in geometry, *Lipschitz and Hölder maps* come up in analysis, *continuous maps* underlie topology, *Borel maps* are essential in the measure theory.

Morphisms are hardly sufficient by themselves to detect, express and quantyfy *similarities* between different objects in the context of ergosystems; the category theoretic structure must be complemented by geometric structure(s)., e.g. as follows.

Suppose, there is a notion of distance between two morphisms $X \to Y$ in our category. This is the case, for example if the objects are metric spaces themselves, where the distance between maps f_1, f_2 from X to Y can be defined, e.g. as the maximum (or supremum) over all points x in X of the Y-distances between $f_1(x)$ and $f_2(x)$ in Y. (There are more interesting distances which use the geometry of X as well as of Y.)

Then, following the guidelines of the category theory, define the distance between X and Y, as the minimal δ, for which there exist morphisms $f : X \to Y$ and $g : Y \to X$, such that

$\circlearrowright_\delta$: the composed morphisms $f \circ g : Y \to Y$ and $g \circ f : X \to X$ are δ-close to the respective identity morphisms.

However, if you apply this to the category of metric spaces and *isometric* maps, this does not work, since, typically, there are no morphisms from X to Y at all. (The pools of morphisms in many categories are too shallow to accommodate such a definition.) And if you significantly enlarge the sets of morhisms, e.g. allowing continuous maps, then the distance may become zero for quite dissimilar X and Y.

This can be remedied with a quantified notion of an "approximate morphism", say an *ε-morphism* between spaces. One may say that X and Y are $(2\varepsilon + 2\delta)$-*close* if there are ε-approximate morphisms f and g, which satisfy the above $\circlearrowright_\delta$.

Accordingly, one may define a distance between metric spaces (honestly, between isometry classes of spaces) with ε-morphisms $f : X \to Y$ which are understood as the (possibly discontinuous) maps f where the metric on X, regarded as a function $d(x_1, x_2) = d_X(x_1, x_2)$, is ε-close to the function induced by f, i.e. to $d_Y(f(x_1), f(x_2))$, where the latter ε-closeness is gauged with some metric in the space of functions d in the variables x_1 and x_2, e.g.

$$dist_{sup}(d_1, d_2) = \sup_{x_1 x_2} |d_1(x_1, x_2) - d_2(x_1, x_2)|.$$

The structures we encounter in ergo-modeling come in discrete combinatorial dressings, something like graph theoretic ones. These bring along varieties of metrics within our objects as well as in the spaces of objects; identifying the best metric(s) for assessing similarities between ergo-objects will be our essential concern.

2.11 Statistics, Probability, Improbability, Prediction and Entropy.

Prediction depends on structural modeling of numerical distributions of repetitions of similar events.

In physics, the resulting structures can be often reperesented by probability measures on some configuration (or phase) spaces, but, for example, a probability measure on the "sets of sentences" is a poor model for a languge. The following shows what goes wrong.

"A six foot man in blue underwear entered a room with five windows and took off his glasses".

Can you assign to this sentence, let it be only approximately, some probability p on the basis of the frequencies of the following fragments of this sentence as they appear on Google?

$$[a]_G \approx 10^9, \qquad [man]_G \approx 10^8, \qquad [\textit{six foot man}]_G \approx 10^4,$$
$$[\textit{a man in blue underwear}]_G \approx 5, \qquad [\textit{man entered a room}]_G \approx 10^5,$$
$$[\textit{room with five windows}]_G \approx 10^4, \qquad [\textit{took off his glasses}]_G \approx 5 \cdot 10^5.$$

Does the probability p of the full sentence satisfy $10^{-30} \leq p \leq 10^{-15}$?

Do the Google numbers

$$[\textit{man in a blue coverall}]_G \approx 5 \cdot 10^3 \text{ and } [\textit{took off his coverall}]_G = 3$$

tell you whether p increases (decreases) if you replace both or either of the two words "underwear" and "glasses" by "coverall"?

Does p equal, let it be approximately, the product of the probabilities of the sentences:

A. "A six foot man in blue underwear entered a room and took off his glasses".

B. "There were five windows in the room".

(Notice that

$$[\textit{there were five windows in the room}]_G \approx 5 << 10^4 \approx [\textit{room with five windows}]_G.)$$

Would you be better off with evaluation of the probability of the *actual event* described by this sentence?

To understand the nature of the difficulty let us look separately at the two key players in the probability theory: *events* and *numbers.*

Probability is a *number* p in the interval $0 \leq p \leq 1$ assigned to an *event* e where the three relevant ingredients of the number structure are:

Add – *Summation*: The probability of occurrence of *at least one* out of several "mutually exclusive" events equals the *sum* of the probabilities of these events.

Ord – *Order*: If an event e_1 "implies" e_2 , i.e. e_1 is necessarily accompanied by e_2, then $p(e_1) \leq p_2$.

Prod – *Multiplication.* The probability of simultaneous occurrence of several "mutually independent" events equals the *product* of the probabilities of these events.

In practice, one isolates a particular pool of events where "exclusive", "implies" and, most essentially, "independent" make certain sense regardless of any idea of probability, and then one tries to assign probabilities to events in a consistent way.

Add and **Prod** allow numerical computations where the eventual decisions/predictions are based on **Ord**: the event with the *greatest* probability will be our first choice to accept (or to reject).

But how does one assign numbers to events? What are criteria for a proper assignment? When does such an assignment make any sense at all?

Some problems are inherent in numbers; they may be too small to have any "real meaning", but this is not as serious as it looks: statistics in physics is a witness to this.

What is more serious is that sentences are not quite "events" of the probability theory – they are not about "exclusive", "implies" and/or "independent".

Our answer, vague at this stage, is that the assignment $e \mapsto p$, whichever e and p stand for, must "naturally transform" (combinatorial like) "e-structures" to simpler "p-structures", where the latter have something to do with addition, multiplication and order.

Ideally, this assignment should be a *(natural) functor* from "the category of (pools of) e's" to a "category of p's" but this ideal is not achieved even in pure mathematics: the probability theory has been escaping the embrace of the category theory so far (although *stochastic categories*, where the sets of morphisms between objects, $M_{12} = Mor(o_1, o_2)$ carry probability measures for which the maps $M_{12} \times M_{23} \to M_{13}$ are measure preserving, are common in "randomized combinatorics".)

The most frequently used functoriality, e.g. in the (quantum as well as classical) statistics amounts to (quasi)invariance of probability measures under symmetry groups. However, this is not sufficient, not even in the framework of the classical statistical mechanics, which is based on the *canonical* Boltzmann-Gibbs distributions rather than on the naive *microcanonical* ones, where "canonical" are distinguished by being *multiplicative* under "unions" of *uncoupled* systems. Furthermore, *grand canonical ensembles* reflect *non-isomorphic* "morphisms" in the "category" of statistical systems.

But the symmetry driven statistics is severely limited when applied, for instance, to such systems $S^\wr$ as protein molecules, since these $S^\wr$ do not have sufficient group theoretic symmetries of their own and since, when such an $S^\wr$ is taken as a part of larger (unconstrained) symmetric system $S^\bigcirc$, it makes too small a part of $S^\bigcirc$ to inherit its symmetries due to a large array of constrains which define $S^\wr$ within $S^\bigcirc$ (such as the the spacial constrains on the amino acid residues in a protein molecule).

Another matter of our discontent with the classical probability is the concept of the "*universal* probability measure space" associated to every system under consideration. This goes back to Kolmogorov's 1933 foundational paper on probability, where events are modeled, by subsets e in a measure space X, say, represented by the segment $[0,1]$ or by the square $[0,1] \times [0,1]$ and where the probability $p(e)$ is represented by the measure (length in $[0,1]$ and area, if X

is the square) of e.

These "universal spaces" are as arbitrary (non-functorial) as as the "universal domains" in Andre Weil's 1946 foundation of algebraic geometry, but, unlike the latter, the measure theoretic formalism has not undergone the functorialization reform.

In reality, all probabilities are *relative* or *conditional* ones. For example, the absolute numbers N of occurrences of events, e.g. of particular words and phrases on Google, even normalized by a common denominator, make little sense. The "meaningful" numbers r are the ratios of *appropriate* products of such N, e.g. $r = \frac{N_1 N_2}{N_1' N_2'}$, where N_1 and N_2 are the numbers of occurrences of two phrases on Google which make together certain sentence s and where N_1' and N_2' are such numbers for another division of the same s into two phrases.

A more more elaborated example is $r = \frac{N_{123} N_2}{N_{12} N_{23}}$, where an s is divided into three fragments, where N_{123} is the occurrence number for the whole s and where N_{ij} are these numbers for joint $i\&j$-fragments.

Fractions of this kind must have equal numbers of terms in numerator and denominator (this alone would not exclude questionable $r = \frac{N_{12}^2}{N_1 N_2}$) and equal multiplicities of the indices. A useful fractions of a different kind is N_{12}/N_{21} counting pairs of consecutive words "2 after 1" against "1 after 2".

(These fractions make one think of probabilities represented by *rays in the cone* $\mathbb{R}_+^2 \subset \mathbb{R}^2$ of pairs of non-negaitive numbers and in associated cones in tensor product of Euclidean spaces. The fractions are turned into "probability *numbers*" by the *normalization* map from the set P_+ of these rays, which is represented by the planar segment of pairs of non-negative numbers $(x_1, x_2) \in \mathbb{R}_+^2$ satisfying $x_1 + x_2 = 1$, onto the "number segment" $[0,1]$. This normalization does not come for free: the set P_+ of rays in $\mathbb{R}_+^2$ is symmetric under the action by the multiplicative group of real numbers $r > 0$ for $(x_1, x_2) \mapsto (rx_1, r^{-1}x_2)$, where this symmetry is obscured by the language of "probability numbers" $0 \leq p \leq 1$.)

Abstractly, the conditional probabilities are defined in the category $\mathcal{L}$ of *Lebesgue-Rokhlin (probability) measure spaces* and *measure preserving* maps via *(categorical) fibered products*, also called *pullbacks of morphisms* and/or *Cartesian squares*.

Recall that every Lebesgue-Rokhlin probability space is isomorphic to disjoint union of a segment $[0, m_0]$, $0 < m_0 \leq 1$, and at most countably many *atoms* x_i, i=1,2,..., of weights $m_i = p(x_i) > 0$, such that $\sum_{i=0,1,2,...} m_i = 1$.

A map $f : X \to Y$ is called *measure preserving* if

$$measure_X\left(F^{-1}(Y_0)\right) = measure_Y(Y_0) \text{ for all } Y_0 \text{ in } Y.$$

The existence of fiber products in $\mathcal{L}$ was proven by V. A. Rokhlin (in terms of *measurable partitions*) more than half-century ago, where the *fiber product* of two *objects X and X' over Y*, i.e. of two morphisms $f : X \to Y$ and $f' : X' \to Y$ (in an arbitrary category) is an object $(X^\times, f^\times)$ over Y, often denoted $X \times_Y X'$, with two morphisms $g : X^\times \to X$ and $g' : X^\times \to X'$ which have the same formal properties (1) and (2) as the projections of the Cartesian product of two sets onto its components, $g_1, g_2 : A \times A' \to A, A'$, with f and f' being the constant maps to the one point set in this case:

(1) commutativity of the square diagram of the four arrows: $f \circ g = f' \circ g' = f^\times$;

(2) universality of $(X^\times, g, g')$ for (1): if $(X^\times_\circ, g'_\circ, g'_\circ)$ satisfy (1) then there is a unique morpism $h : X^\times_\circ \to X^\times$, where all obvious arrows commute.

In other words, $X \times_Y X'$ in a category $\mathcal{C}$ is the Cartesian product in the *category* $\mathcal{C}_Y$ *of objects over* Y, i.e. the category of morphisms f from all X to a fixed object Y in $\mathcal{C}$, where morphisms in $\mathcal{C}_Y$ are *commutative* triangular diagrams of morphisms $f, f' : X, X' \to Y$ and $f_Y : X \to X'$, where "commutativity" means that $f = f' \circ f_Y$.

Notice that fibered products can be composed, where the patterns of iterations of compositions essentially correspond to the above $N.../N...$-ratios in statistical numerics.

The Lebesgue-Rokhlin category $\mathcal{L}$ can be seen as a natural/canonical *categorization* of positive real numbers, similarly to how the category of finite sets is derived from the Peano arithmetic of finite ordinal numbers. Picturesquely, each number $p \geq 0$ is represented by a pile of sand of mass (measure) p, where morphisms correspond to bringing several piles together and adding up their masses.

On the other hand, it is easy to see that $\mathcal{L}$ equals the "categorical metric completion" of the subcategory $\mathcal{FL}$ of finite sets X in $\mathcal{L}$ where the "probability morphisms" in $\mathcal{FL}$ are the maps $f : X_1 \to X_2$, such that the cardinalities of the pullbacks $Y_1 = f^{-1}(Y_2)$ of all subsets $Y_2 \subset X_2$ satisfy

$card/card$
$$\frac{card(Y_1)}{card(X_1)} = \frac{card(Y_2)}{card(X_2)}.$$

In order to speak of limits and completions, one needs a notion of distance between morphisms and one takes the Hamming metric normalized by $1/card(X_1)$ for this purpose. Besides, to have sufficiently many limits, these "pure" morphisms need to be relaxed to *ε-morphisms* where $card/card$ is satisfied only up to an $\varepsilon > 0$.

Then one passes to the limit category and sends $\varepsilon \to 0$; thus, all general properties of $\mathcal{L}$ follow from the corresponding properties of (ε-morphisms in) $\mathcal{FL}$ by obvious (but essential) continuity of these properties.

(What the measure theoretic setting does for probability, amounts, from a certain point of view, to putting a particular emphasis on this continuity by calling it *countable additivity* of measures.)

In simple words, the approximation of $\mathcal{L}$ by $\mathcal{FL}$ corresponds to subdividing piles of sands into *equal* sub-piles, where the equality may be only approximate in a presence of "atoms" – indivisible grains of sand.

(Warning: a general *invertible* measure preserving transformation f of the segment $[0,1]$ is *not* a limit of permutations of subsegments $[n/N, (n+1)/N]$, $n = 0, ..., N-1$; however, whether f is invertible or not, *it is* a limit of measure preserving maps sending each $[n/N, (n+1)/N]$ to some $[m/M, (m+1)/M]$ where M may be *much smaller* than N.)

The approximation of probabilities by ratios of (large) integers is implemented in statistics by counting relative frequencies of recurrent events and passing to the limit with longer and longer chains of "experiments". In either case, probability can be seen as a "real number" interpreter of the "count" language which translates "independence"⇒"multiplicativity".

However, blind counting, even augmented by probabilistic conditioning, is not adequate for the study of ergosystems (and of biological systems in general), where real numbers systematically come in a variety of non-numerical structural dressings and where the "structure of this variety" is more subtle than what you find in physics, for example. (One rarely hesitates in physics which experimental numbers can be added and which multiplied, but biological or linguistic "events" come with less obviously identifiable flavors, where their frequencies need to be recorded on different, albeit interconnected, numerical scales.)

Besides, ergo-learning depends on "structurally significant" (often amazing) *rare events*, where the "structurally significant" (as well as "amazing") condition is not quite like the conditioning in the probability theory.

For example, the boundary of a visual image, albeit it has negligible "probability", is the most essential feature of this image. Similarly, if a randomly looking sequence of, say 20, letters appears twice in a 10^5-letter text, it can not be brushed off as a statistical fluctuation.

Our models of ergo-learning will be based on numbers delivered by statistics, but "ergo-assimilation" of these numbers will be not quite probabilistic. An ergolearner, (as all people are) is on constant vigil for rare significant "events", which are recognized on the basis of the prior statistical records of what is "probable"; then the deserving improbable (and some probable) patterns of events are inserted somewhere between probabilistic and categorical wheels incessantly rotating in the structure learning mechanism.

An archetypical example of prediction by "improbability" is that of a coming rain upon hearing remote rumbles of thunder. I guess, a baby animal, who has never lived through a thunderstorm, would run for cover at the first sounds of it, exactly because this rumble contrasts with the regular statistics of natural sounds previously heard and recorded in the animal brain.

Simple and most useful for us Lebesgue-Rokhlin spaces are *finite* probability spaces X where each point x in X carries a weight $p(x) > 0$, such that

$$(\Delta^{N-1}) \qquad \sum_x p(x) = 1.$$

This equation defines an $(N-1)$-simplex for $N = card(X)$ in the space (cone) $\mathbb{R}_+^X = \mathbb{R}_+^N$ of positive functions $p : X \to \mathbb{R}$.

A map $f : X \to Y$ is measure preserving if the sum of weights of all x contained in the pullback $f^{-1}(y)$ of each y in Y (i.e. of x, such that $f(x) = y$) equals $p(y)$ in Y.

The product of two spaces $X \times X'$ is made of atoms (x, x') of weights $p(x, x') = p(x)p(x')$.

The fiber product $X \times_Y X'$ consists of those (x, x') for which the implied maps $f, f' : X, X' \to Y$ satisfy $f(x) = f(x')$ and where $p(x, x') = \frac{p(x)p(x')}{p(y)}$ for $y = f(x) = f(x')$.

(Verification of the properties of the fiber product amounts here to a multiplication table kind computation. What is amusing, however, is that the result of this computation, no matter how simple, is "predicted" by the formalism of the category theory.)

Besides probability, there is another real number which extends the idea of cardinality, namely the *Shannon (Boltzmann) entropy* of a probability measure $p(x)$ on a finite set X

$\mathbf{B}_{log}$ $$-\sum_{x\in X} p(x)\log_2 p(x),$$

that has a remarkably simple characterization which, apparently, motivated Boltzmann's introduction of it to physics.

$\mathbf{B_0}$. If the function $p(x)$ is constant on a subset X_0 of X and it vanishes outside X_0 then

$$ent(p(x)) = ent(X_0) =_{def} \log_2 card(X_0).$$

(In physical terms, the entropy is the logarithm of the number of states of a system.)

$\mathbf{B}_+$. The entropy is additive for Cartesian products: the entropy of $p(x,y) = p_1(x)p_2(y)$ on $X \times Y$ equals the sum of the entropies of p_1 and p_2. (If two physical systems do not interact, their entropies add up.)

$\mathbf{B}_\infty$. The entropy is continuous; what is essential is that if a distribution p is "almost constant" on a subset X_0 in X and is "almost zero" outside X_0 than the entropy is "close" to $\log_2 card(X_0)$. (This condition, without specification of the "almost" and "close", is too obvious for a physicist to be ever mentioned.)

Observe that the $\mathbf{B}_{log}$-entropy does satisfy these three properties – this is nearly obvious; what is remarkable–this goes back Boltzmann and Gibbs – these properties *uniquely* define the entropy as well as clarify its "physical" (categorical) meaning.

Indeed, according to $\mathbf{B}_+$,

$$ent(X,p) = \frac{1}{N} ent(X,p)^N \text{ for all } N = 1,2,...,$$

where $(X,p)^N$ is the N-th Cartesian power of $(X,p)^N$ with $p(x_1,...,x_N) = p(x_1)\cdot ....\cdot p(x_N)$.

If N is large, then, by the *law of large numbers* applied to the random variable $\log_2 p(x)$ on the probability space $(X, p(x))$ the function

$$p(x_1,...,x_N) = p(x_1)\cdot ....\cdot p(x_n) = 2^{\sum_i \log_2 p(x_i)}$$

is "almost constant" (in the required sense) on a subset $X_0^N = X_0^N(p)$ in X^N and is almost zero away from it; thus,

$$ent(X,p) = \frac{1}{N}\lim_{N\to\infty} log_2 card(X_0^N)$$

by $\mathbf{B}_0$ and $\mathbf{B}_\infty$. (This *Gibbs' tensorisation trick* as amazingly efficient in reducing a general function on a space to the indicator function of a subset, in a larger space.)

Two other relevant (and quite simple) properties of the entropy are:

(1) entropy *vanishes* if and only if $p(x) = 0$ for all x in X except a single x_0,

(2) entropy is *maximal* if $p(x) = 1/card(X)$ for all x in X, where $ent = \log_2 card(X)$.

In the case (1) you can predict that $x = x_0$ with the 100% assurance while in the case (2) your prediction does not worth anything. In general the entropy can be interpreted as a measure of how well you can "predict" which point "event" x comes up on the basis of $p(x)$.

Thus, minimization of entropy is similar to maximization of predictive power by an ergo-learner; however, the two concepts diverge beyond the initial stage of our models of learning processes.

(The entropy function is also distinguished by its hidden orthogonal symmetry, see 3.6, which apparently originates in the *canonical quantization* and the *moment map*; but all this does not play a visible role in our models of ergo-learaning.)

2.12 Mathematics of a Bug on a Leaf.

Think of a bug crawling on a leaf or of your eye inspecting a *green* spot on, say, a *brown* background.

We assume (being unjust to bugs) that all the bug can directly perceive in its environment are two "letters" a_1 and a_2 – the colors (textures if you wish) of its positions on the leaf, where the bug has no idea of color (or texture) but it can distinguish, say, green a_1-locations from the brown a_2.

The four "words" our bug (eye) creates/observes on contemplating the meaning of its two consecutive positions are a_1a_1, a_1a_2, a_2a_1 and a_2a_2. But can the bug tell a_2a_1 from a_1a_2? Which of these words are similar and which are not? Is a_1a_1 is "more similar" to a_1a_2 than to a_2a_2?

These questions are, essentially, group theoretic ones. (Tell this to a bug, and it would as much surprised as Monsieur Jourdain upon learning he spoke prose.)

There are two distinguished (commuting involutive) transformations acting on the set of these four words:

1.*Alphabetic symmetry*: Switching the colors, $a_1a_1 \leftrightarrow a_2a_2$ and $a_1a_2 \leftrightarrow a_2a_1$.

2. *Positional symmetry*: Interchanging the orders of the letters, $a_1a_2 \leftrightarrow a_2a_1$, with no apparent action on a_1a_1 and a_2a_2.

(Recall, that a transformation T is an involution if $T \circ T(x) = x$ for all x, and T_1 commutes with T_2 if $T_1 \circ T_2 = T_2 \circ T_1$, where $T_1 \circ T_2$ denotes the composition: $T_1 \circ T_2(x) =_{def} T_1(T_2(x))$.)

(If you tried to explain this to a bug, it would be much annoyed since:

1. Traversing a leaf is an *art* rooted in a *holistic intuition* of insects which can not be reduced to the abstract nonsense of algebra.

2. Insects have been managing pretty well for the last 300 million years, while the adepts of abstract algebra can not boast a comparably venerable past, no do they have much future on the evolutionary time scale.

3. The group theory is *not useful* in serious matters such as eating a tasty leaf, for a example.

Amusingly, some mathematicians also insist on the superiority of "concrete", "pragmatic", "artistic" and "holistic" approaches to their beloved science. This, however, is not supported by the historical evidence: "artful mathematical pieces" are eventually become incorporated into "abstract" general theories, or else, they become extinct. Thus, for example, the *Wilf-Zeilberger algorithm* relocated exquisite flowers of *hypergeometric identities* form the rain forest of artistic imagination to the dry land of formal reasoning.)

The above transformations do not essentially change the "meaning" of the words: a green square on a brown background is identical, for most purposes, to a brown square on a green background. (I wonder, how fast this "equality" is recognized by different kinds of animals.) Also, the "essence" of a_1a_2 is the *change* of colors as you go from one location to the other, rather than the colors themselves.

(There is yet another involutive transformation: changing the color of the first letter : $a_1a_1 \leftrightarrow a_2a_1$, $a_1a_2 \leftrightarrow a_2a_2$, $a_2a_1 \leftrightarrow a_1a_1$ and $a_2a_2 \leftrightarrow a_1a_2$. This involution together with the positional one generate a non-commutative group with 8 elements in it, called the *wreath product* $\mathbb{Z}_2 \wr \mathbb{Z}_2$; the role this group plays in the life of insects remains obscure.)

Alphabet of Bug's Moves. Our bug (or eye) has a certain repertoire of moves but it knows as little about them as it knows about colors. Imagine, each move being represented by a letter, a "button" the bug may press. As a result, the bug (eye) sees the color of the location this move/button brings it to.

The basic knowledge the bug possesses is the equality/non-equality relation between two moves. Since the bug does not know its own position, the *equality of two moves* from bug's perspective – pressing the *same button* – implies that

the two spacial moves go in the *same direction* (relative to bug's orientation) and at the *same* distance, *regardless of the location* of the bug at the moment it presses the button.

(The eye, unlike the bug "knows" its position s and, in order to repeat a move, it needs to "forget" s. Besides the eye has several independent arrays of buttons corresponding to different modes of eye movements, some of which are rather random.)

This may seem not much to start with; amazingly, this is exactly what is needed for reconstruction of the *affine structure* of the Euclidean plane: this structure is uniquely determined by distinguishing particular triples of points x_1, x_2, x_3, namely those which lie on a line with x_2 being halfway from x_1 to x_3.

Moreover, if the bug can "count" the number of repetitions of identical moves, it can evaluate distances and, thus, reconstruct the full Euclidean (metric) structure of the space, 2-plane in the present case.

Which buttons has the bug (eye) to press in order to efficiently explore the leaf and learn something about its meaning – the shape of the leaf?

The bug (eye) feels good at the beginning being able to predict that the color usually does not change as the bug (eye) makes small moves. (The eye, unlike the bug, can make fast large moves.) But then it becomes bored at this repetitiveness of signals, untill it hits upon the edge of the leaf. Then the bug (eye) becomes amazed at the unexpected change of colors and it will try to press the buttons which keep it at the edge.

(Real bugs, as everybody had a chance to observe, spend unproportionally long time at the edges of leaves. The same applies to the human eyes.)

In order to keep at the edge, the bug (this is more realistic in the case of the eye) needs to remember its several earlier moves/buttons. If those kept it at the edge in the past, then repeating them is the best bet to work so in future. (This does work if the edge is sufficiently smooth on the bug's scale.)

Thus, the bug (eye) learns the art of to navigation along the edge, where it enjoys twice predictive power of what it had inside or outside the leaf: the bug knows which color it will see if it pushes the "left" or the "right" buttons assuming such buttons are available to the bug. (The correspondence "left"$\leftrightarrow$"right" adds yet another involution to the bug's world symmetry group.)

Amazingly, this tiny gain in predictive power, which make the edge interesting for the bug, goes along with a tremendous information compression: the information a priory needed to encode a leaf is proportional to its area, say $A \cdot N^2$ bits on the N^2-pixel screen (where A is the relative number of the pixels inside the leaf) while the edge of the leaf, a curve of length l, can be encoded by $const \cdot l \cdot N \cdot \log N$ bits (and less if the edge is sufficiently smooth). Unsurprisingly, edge detection is built into our visual system.

(The distribution of colors near the edge has a greater entropy than inside or outside the leaf but this is not the only thing which guides bugs. For example, one can have a distribution of color spots with essentially constant entropy across the edge of the leaf but where some pattern of this distribution changes at the edge, which may be hard to describe in terms of the entropy.)

Eventually the bug (eye) becomes bored traveling along the edge, but then it comes across something new and interesting again, the tip of the leaf or the T-shaped junction at the stem of the leaf. It stays there longer plying with suitable buttons and remembering which sequences of pressing them were most interesting.

When the bug start traveling again, possibly on another leaf, it would try doing what was bringing him before to interesting places and, upon hitting such a place, it will experience the "deja vu" signal – yet another letter/word in bug's language.

We have emphasized the similarities between the eye and the bug movements but there are (at least) two essential differences.

1. The eye moves much faster than the bug does on the neurological time scale.

2. The eye can repeat each (relatively large) "press the button" move only a couple of times within its visual field.

On the other hand, besides "repeat", there is another *distinguished* move available to the eye, namely, "reverse". This suggests that the (approximate) back and forth movements of the eye will appear unproportionally often. (Everybody's guess is that the eye would employ such moves for comparing similar images, but I have not check if this has been recorded in the literature.)

Geometric incarnations of "reverse" are more amusing than the affine spaces associated with "repeat". These are known in geometry as *Riemannian symmetric spaces* X (see 3.6) where each "move" issuing from *every* point x in X is "geometrically equivalent" to the reverse move from the same x, i.e., for every x in X, there is an involutive isometry I_x of X the fixed point set of which equals x (i.e. $I_x(y) = y \Leftrightarrow y = x$).

The simplest instances of such X are the Euclidean, hyperbolic and spherical spaces, e.g. the 2-sphere of the visual field of the eye but there are other spaces with more elaborate and beautiful symmetric geometries (see 3.6.)

But the problems faced by out bug are less transparent than evaluating a metric in a *given* space. Indeed, pretend that you – a mathematician – are such a bug sitting at a keyboard of buttons about which you know nothing at all. When you press a button, either nothing happens (the color did not change) or there is a blip (indicating the change of the color).

Can you match these buttons with moves on the plane and the blips with crossing the boundaries of monochromatic domains?

What is the fastest strategy of pressing buttons for reconstruction the shape of a domain?

The answer depends, of course, on the available moves and the shapes of the domains you investigate: you need a rich (but not confusingly rich) repertoire of moves and the domains must be not too wild.

What you have to do is to create a language, with the letters being your buttons(+blips), such that the geometric properties of (domains in) the plane would be expressible in this language of sequences of (pressing on) buttons marked by blips. If in the course of your experiments with pressing the buttons, you observe that these properties (encoded by your language) are satisfied with the overwhelming probability, you know you got it right.

But what the bug (eye) has to do is more difficult, since, apparently, there is *no a priori idea of spacial geometry* in bug's brain. Bug's (eye's) geometry is, essentially, the grammar of the "button language". Thus, bug's brain (and an ergobrain in general) can not use a strategy tailored for a particular case, but may rely only on some *universal* rules, as the real bugs (eyes), we believe, do. The success depends on the relative simplicity/universality of the plane geometry, more specifically on the group(s) of symmetries of the plane. (This symmetry is broken by "colored" domains in it, and, amusingly, *breaking* the symmetry makes it perceptible to an "observer" – the bug or the eye at the keyboard.)

Apparently, the bug is able to make an adequate picture of the world, because the mathematical universality of bug's strategies matches the universal mathematical properties of the world.

2.13 Automata and Turing Machines.

Let us paraphrase our "a bug on a leaf" story in the language of *finite state automata.*

Denote by B the finite set of possible *moods* or *internal states* b of a bug, which, roughly, correspond to the above "buttons" which define bug's moves.

(This "finite" gives a wrong idea of *exponentially many* different states of a bug. But it may be not so absurd to assume that the sets of moods of insects which visibly influence their behaviors are listable.)

The *spacial positions* of a bug B in its environment are represented by another set, say S, the points s of which may encode the spacial orientations as well as the locations of the bug. These points carry some marks $a = a(s)$ on them, e.g. patches of different colors or odors perceptible by the bug. The *alphabet* of the bug – the set of these "colors" or "letters" is denoted by A.

The set B is given the structure of *an A-colabeled graph*, which means that B serves as the vertex set of a directed graph where the edges are "co-labeled" by letters from A via a *colabeling map* $\overrightarrow{\beta}(a,b)$ with values in the set $\overrightarrow{E}(b)$ of the edges-arrows issuing from b: if the bug in the mood b "sees" a it "chooses" the edge $\overrightarrow{\beta}(a,b)$ issuing from b.

The edge valued map $\overrightarrow{\beta}$ defines a B-valued map, denoted β which sends (a,b) to the terminal vertex, of $\overrightarrow{\beta}(a,b)$; thus $b_1 = \beta(a,b)$ equals the terminal vertex of the edge-arrow $\overrightarrow{\beta}(a,b)$ issuing from b, where this is interpreted as the internal move of the bug from the state b to $b_1 = \beta(a,b)$ when the bug perceives a.

The space S, in turn, is given the structure of a B-*colabeled* graph. Here the set $\overrightarrow{E}(s)$ of edges issuing from s represents the changes of position available to the bug located at s, where the choice of such a move is determined by the mood b of B. The corresponding co-labeling map $B \times S \to \overrightarrow{E}(s)$ is denoted $\overrightarrow{\sigma}(b,s)$ and $s_1 = \sigma(b,s)$ stands for the terminal vertex of the arrow $\overrightarrow{\sigma}(b,s)$.

Thus, the state of a bug is represented by a pair (b,s), where b is the mood of B and s is the spacial position of B. The bug moves, internally and externally, according to the letters a written on S by the rule

$$(b,s) \rightsquigarrow (b_1,s_1) \text{ for } s_1 = \sigma(b,s), \text{ and } b_1 = \beta(a_1,b) \text{ where } a_1 = a(s_1).$$

(Colabeled graphs can hardly serve as adequate models of "ergo-bugs" but some of theirs purely mathematical representatives, such as *Cayley graphs* of finitely generated groups S are amusing in their own right. Besides, there are natural morphisms between controlled graphs as well as between the colabeled graph models $(B,S,A,\overrightarrow{\beta},\overrightarrow{\sigma})$ of "crawling bugs" which make a descent category. Also notice that the *outcomes* of bug's movements depend only on the maps β and σ, rather than on $\overrightarrow{\beta}$ and $\overrightarrow{\sigma}$; the reason for introducing these $\overrightarrow{\cdots}$ will become clearer later on.)

Turing Bugs. A *T-bug*, unlike the above automaton-bug, is able to modify its environment, e.g. it can leave a particular scent at the location he has visited or to erase a scent (color) which may influence the behavior of the bug when it comes across the trace of its own scent. Being finite itself, a T-bug has the infinite resource of the environment at its disposal for storing its memories.

The "modification of color" is encoded by a function $\alpha : b \mapsto a$; thus, when B moves from s to another location, it assigns the letter $\alpha(b)$ to s, where b indicates the moodof B at s.

In sum, the full state of our Turing system is described by $t = [b,s,a(s)]$, where $a(s)$ is an A-valued function on S, i.e. an element of the set A^S of A-

valued functions on S. The behavior of the T-bug is depicted by a map $\mathcal{T}$ of the space $T = B \times S \times A^S$ into itself that is

$$\mathcal{T} : t_0 = [b_0, s_0, a_0(s)] \rightsquigarrow t_1 = [b_1, s_1, a_1(s)],$$

where b_1 and s_1 are defined as earlier,

$$b_1 = \beta(a_0(\sigma(b_0, s_0), b_0),\ s_1 = \sigma(b_0, s_0),$$

and where the function $a_1(s)$ is defined by

$$a_1(s = s_0) = \alpha(b_0) \text{ and } a_1(s \neq s_0) = a_0(s).$$

Turing Computation. Denote by $t_0, t_1, ..., t_i, ...$ the $\mathcal{T}$*-orbit* of a point t_0 in T, obtained by repeatedly applying $\mathcal{T}$ to $t_i = [b_i, s_i, a_i(s)]$, starting from $t_0 = [b_0, s_0, a_0(s)]$, i.e. $t_i = \mathcal{T}(t_{i-1})$. If this sequence t_i eventually stabilizes, i.e. $t_{i+1} = t_i$ for $i \geq i_{stop}$, we regard the $a_\infty = a_\infty(s)$ component of the resulting stable state

$$t_\infty = [b_\infty, s_\infty, a_\infty(s)] =_{def} [b_i, s_i, a_i(s)], i \geq i_{stop}$$

as

the final result of the (B, b_0, s_0)*-computation applied to the function* $a_0 = a_0(s)$,

$$a_\infty = \mathcal{T}^\infty(a_0) = \mathcal{T}^\infty_{b_0 s_0}(a_0).$$

(Observe that t_∞ is a *fixed point* of $\mathcal{T}$, i.e. $\mathcal{T}(t_\infty) = t_\infty$.)

If the sequence does not stabilize, we write $\mathcal{T}^\infty_{b_0 s_0}(a_0) =$ "?"

Turing Machines. A T-bug is called a *Turing machine* if

• S equals the set of integers. (Often, one takes the set of *positive* integers but this is notationally less convenient.)

•• The set $\overrightarrow{E}(s)$ of edges-arrows issuing from s consists of the two arrows: $s \to (s+1)$ and $s \to (s-1)$ for all numbers s in S.

••• The map σ is $\mathbb{Z}$*-invariant.* Thus, σ is determined by a map from B to the two element set $\overrightarrow{E}(0) = \{+1, -1\}$ (the ± 1 elements of which the bug can "interpret" as "left" and "right" with respect to its position s, consistently at all s.).

Usually, when one speaks of computations, one restricts to *eventually constant* functions $a = a(s)$, i.e. constant for $|s| \geq N = N(a)$. Then the input of the computation is represented by a member of the *finite* set $A^{\{-N, -N+1, ..., N-1, N\}}$ of cardinality $card(A)^{2N+1}$. However, the overall complexity of the computation *can not* be "effectively estimated" by the numbers $card(A)$ and N.

Namely, there exists a finite "bug" B, which runs, say, on the 2-letter alphabet A and certain $\overrightarrow{\beta}$ and $\overrightarrow{\sigma}$, such that the computation stabilizes (terminates) for all eventually constant functions $a = a(s)$, where the time i_{stop} needed for such computation is, obviously, bounded by some function $stop(N)$ for $N = N(a)$.

But this *stop*-function may grow faster than anything you can imagine, faster than anything expressible by any conceivable formula – the exponential and double exponential functions that appeared so big to you seem as tiny specks of dust compared to this monstrous *stop*-function. (In fact, there are lots of B, $\overrightarrow{\beta}$ and $\overrightarrow{\sigma}$ with this property as it follows from the existence of *universal Turing machines* and insolvability of the *halting problem.*)

This happens, since the Turing machines, despite their apparent speciality and innocuous finitery outlooks, are as general and as anything in the world, they harbor unlimited layers of complexity.

(These "machines" guard the border of mathematics: if your problem, even remotely, is reminiscent of $stop(N)$, you'd better look for another problem. Ergosystems display similar attitudes – they shy away from $stop(N)$. This, necessarily, makes the domain of inputs, to which they respond with "?", greater than that for the Turing machines, where some "ergo-"?" may be followed by "emergency strings"...!".)

Notwithstanding their "unlimited generality", Turing machines are not flexible enough for *properly* modeling even humble bugs on 2-*dimensional* leaves. Mind you, everything describable in any language can be, "in principle", modeled by a Turing machine, but the catch is that such "modeling" is highly non-canonical and arbitrary, and there is no(?) Turing Galois theory to keep track of this arbitrariness.

Turing modeling irrevocably erases symmetries in the systems to which it applies and brings along a huge amount of redundancy, e.g. by enforcing linear order structures on, a priori, unordered sets, such as the set of pixels on the computer screen.

There are many formalisms "equivalent" to Turing machines, but they suffer similar shortcomings. Besides, such an equivalence, e.g. the equality of the class of functions defined by Turing machines and the class of *recursive functions* may be tricker than it seems, since it depends on the positional (e.g. binary) representation of integers (and/or programs) in the Turing machines.

Thus, a *general* definition of such equivalence/equality needs a preliminary definition of a class of "admissible" representations/interpretations of integers. (If this does not convince you, try to make a *general* definition of "equivalent" and see if a computer program would recognize your equivalence between *the* λ-*calculus* and *Conway's Game of Life*, for example.)

The two ergo-lessons one may draw from Turing models are mutually contradictory.

1. A repeated application of a simple operation(s) may lead to something unexpectedly complicated and interesting.

2. If you do not resent the command "repete" and/or are not getting bored by doing the same thing over and over again, you will spend your life in a "Turing loop" of an endless walk in a circular tunnel.

3 Structures in Life.

3.1 Connectivity, Trees and Evolution.

3.2 Trees, Sequences and Mutations.

3.3 Polypeptides and Proteins.

3.4 Protein Spaces and Protein Counting.

3.5 Proteins Energy Landscapes.

3.6 Sweet Learning by Bacteria.

3.7 Tongue, Nose and the Space of Smells.

3.8 One+One for Hearing, Three+One for Vision

3.9 Dimensions of Motion.

3.10 Universality in Biology.

4 Syntactic Structures.

Learning is an interactive process: a system/program learns by exchanging signals with an environment, e.g. by communicating/conversing with another (ergo)system.

We limit ourselves in this chapter to learning in a "frozen environment" which may be represented by a *library of texts* – "arrays of symbols", organized into books, pages and/or *strings* of letters and words.

Before engaging into describing specific learning strategies employed by ergosystems, let us explain what the result of learning – called *understanding* – is supposed to be.

4.1 Information, Redundancy, Understanding and Prediction.

The most essential aspect of "understanding" is "compression of information":

The *understanding* of a text, is, first of all, the structural understanding of *redundancies* in this text, rather than of "useful information" encoded in the text.

For example, if we deal with natural languages, we expect a significant "compression", say from 10^{10} bytes to 10^7 bytes, without a substantial loss of *structural* components in texts, where "structure" means the structure of redundancy. One can not much compress the "useful information" without loosing this "information" but if we can "decode" the structure of redundancy it can be encoded more efficiently.

The predictive power of (natural) sciences, especially of physics, relies on much stronger compression of data. For example the library of locations of planets in the sky at different times (so useful for ancient hunters, herders, and farmers who understood very little of it) can be compressed to a few lines of (classical or relativistic) laws of motions plus initial conditions. Understanding science is, essentially, understanding the structure of this compression.

But if your understanding of the structure of a 100-byte fragment in a 10^{10}-byte library, depends on the full 10^{10}-byte memory, then your "understanding" is virtually zero.

It follows that if a text, such as a telephone directory, for instance, is essentially non-redundant, then there is nothing to understand in there. (Yet, non-trivial redundancy is present in the "extended text" including the records of the eyes and hands movements of somebody using the directory. Here, indeed, there is something to understand.)

(One may argue, however, that there is much to understand about fully non-redundant, i.e. random, sequences – the whole body of the probability theory is concerned with the structure of randomness. We shall see later on what kind of redundancy makes the probability theory more interesting than a telephone directory.)

What are criteria for evaluating "understanding"?

Apparently, there is no such thing as "absolute understanding"; however, "understanding programs" can be partially(!) ordered according to their "predictive abilities", where the two essential characteristics of a prediction are its *specificity* and *success frequency.*

For example, you are asked to make a list of possible adjectives which would complete the sentence "The crocodile was ... ." How to choose such a list? What should you value more: a short specific list, say of 20 words, which would cover 90% of all such sentences (e.g. found by Google) or a long one – 100 words with 99.9% rate of success?

Also you can predict quite a bit with only "...ile" left of "crocodile", if you know that the storyteller just returned from Egypt and that he is prone to exaggerations.

Ideally, you want to make predictions of all kinds, but this is out of reach, since the set of "all lists" is exponential. Yet, you can represent "the full prediction" by the probability measure on words (or pairs of words) according to the their relative frequencies on Google.

More generally, "predictions", at least in a frozen environment, can be represented in terms of correlations between (and within) "structural patterns" (yet to be defined) in the strings in the library. (But this is not quite so for interactive dialogs (experimental sciences), where you question and/or predict responses of your interlocutor (nature) to the strings (experiments) of your own design, see ???.)

We postpone a description of structural patterns of redundancies in texts as well as structural organization of ergosystems enlisting these patterns till sections ??? and indicate below three essential characteristics of "understanding programs" Pro besides their "predictive powers".

1. *Memory* – the total size of Pro.

2. *Energy* used by Pro for making a given prediction Π, which is measured by the total number of "elementary operations" needed to make the prediction.

3. *Time* needed for Pro to predict Π, which is measured by the length T of the maximal chain of consecutive operations which lead to Π. (Clearly, $energy \geq time$ and the ratio $energy/time$ tells you how many computations go in parallel.)

A final, more subtle criterion which is harder to formalize is "mathematical elegance" of Pro, which, inevitably, depends on the class of texts the program applies to; we return to this in ???.

In the end of the day, we wish to have a "competent ergosystem", which would be applicable to ≈ 100 byte strings (i.e. finding correlations in these) in 10^{10}-10^{12}-byte libraries Lbr, which would be implemented by programs $Pro = Pro(Lbr)$ compatible with the computational and memory resources of modern computers (say, with memory 10^7-10^{10} bytes, energy 10^6-10^9 and time 10^3-10^7) and which would be comparable in its predictive power to that of a competent human ergobrain.

We do not even dream of directly writing down such a programs Pro, but we shall suggest in ??? a design of a universal learning program lea (a model of a baby's ergobrain) which transform $Lbr \mapsto_{lea} Pro$, i.e. builds Pro on the basis of a given library Lbr.

Prediction Evaluation Tests. The way you rate the performance of a program depends on a test. Different tests will give different evaluations of programs, but, even for a fixed test, some programs may be hard to compare. Here is an example.

Suppose you ask the program to select one of the two words in the sentence "This crocodile (horse, cow or man if you prefer) is ..."

smart/stupid, big/small, greenish/grayish friendly/lucky, clever/fast, elliptic/circular, quiet/playful,...,

where you may or may not allow the answers "none of the two" and/or "I do not know".

If you reward correct answers, then the best ego-strategy of a program is to make random guesses, (with, possibly, "none" for elliptic/circular). On the other hand, a reward indifferent ergo-program will soon become bored/annoyed (especially, if you do not allow "do not know" answer) and will respond with an "emergency string" (as robots do in our stories) after the third or forth of such questions. (This may be not well taken by your school teacher who asks you to name the capitals of remote countries or to tell the genders of nouns in a foreign language.)

The above may look non-seriuos, but designing tests for performance of a program is a serious task, albeit it is much easier than making a program itself.

4.2 Libraries and Dictionaries.

A *library* (representing a language) $\mathcal{L}$, by definition, consists of a *background space* or the *backbone* B of $\mathcal{L}$ and of "elementary carriers of information" – called *letters* – "written" on B.

There is not much structure in letters in *syllabic* writing systems, essentially they are spatially *separate* recognizable entities from a short alphabet (with the capitalization convention for proper names in many languages). The written signs for letters, being conventional, play (almost) no role in the perception of a language; yet, the individuality and separateness of signs are helpful: continuous handwritings and binary transcripts (e.g the digital representation of a dvi file) may be hard to decipher.

The (in)significance/redundancy of the letter-signs can be seen in the fact that a permutation π of the letters (almost) does not change anything in the "meaning" of texts.

This can be quantified for a language $\mathcal{L}$ (unknown to you) by the minimal l such that you can determine π, by studying two different randomly chosen texts of length l in $\mathcal{L}$, where one is written in the letters *as they were* and the other one in the π*-permuted* letter-signs.

(This l will be different, say, for the permutation $a_1 \leftrightarrow a_2$ of the green/brown bug's language from 3.11, for English with 26-letter alphabet and for a permutation of $\approx 5 \cdot 10^5$ Chinese characters, which carry the loads of "meaning" comparable to that for words in English.)

Besides the *alphabetic symmetry* of (the group of) permutations of letters, languages carry some *positional symmetry* coming from their backbones (spaces).

For instance, if you think of a natural language as written on an infinite tape, you may say that its positional symmetry "equals" $\mathbb{Z}$ – the additive group of integers. However, this does not reveal the full symmetry which allows, for example, interchanging certain segments of texts, e.g. some words. (This symmetry does not quite fit into a group theoretic framework, since you *can not unrestrictedly compose* such transformation without eventually loosing the "meaning" of what was written on the tape.)

There is another apparent transformation, inversion of the order on the integers, $z \mapsto -z$. The question it raises (a mathematician's folly from the perspective of a pragmatic linguist) is as follows.

How much does the understanding of a text depends on the *direction* it is being read?

More specifically, is there some *universal* set of rules which, when applied to *every* human language, could reconstruct the direction of speech by looking on strings of given length l?

The phonology, probably, would provide a solution, but we insist here on written languages and seek a direction criterion based on some kind of "information profile" of the flow of signals, something like "predictability (function)" $p(z)$ of a sign (letter, word, phrase) at the position z on the basis of several preceding signs.

Do the functions $p(z)$ for different languages have something in common which would distinguish them from all $p(-z)$?

The group G of the (positional) symmetries of the backbone B (e.g. $G = \mathbb{Z}$ for $B = \mathbb{Z}$), and the group Π of permutations of letters at a *single location* (e.g. at 0 in $\mathbb{Z}$) generate what is called the *wreath product* $\Pi^{\wr B \backslash G}$. It is a non-commutative and very large group: it contains all Π-valued functions on B with *finite supports* (assuming G acts *transitively* on B). This group is not(?) *directly* applicable to languages; yet, it unifies the backbone and alphabetic symmetries in a concise way.

Languages also display some category theoretic features: grammatically acceptable *inclusions* of phrases into sentences and making *summaries* of texts are kind of (syntactic) *injections* and (semantic/pragmatic) *surjections* respectively, while translations of texts from one language to another look like functors between languages. (Here again, one has to broaden the category theoretic formalism in order to properly account for imprecision and ambiguity of "linguistic transformations".)

The "letter model" needs a non-trivial adjustment for vision and hearing (not to speak of olfaction and somatosensory systems), where the relevant background space is never a problem (see ???) but where there are no distinct letters: elementary visual and acoustical patterns are "smears of letters" with no sharp boundaries between them, such as phonemes in human speech.

But this is not so bad for written languages, where the basic (but not the only) backbone is that of linear order on places/locations of (certain) letters. Besides that, the *written space* (backbone) is *fragmented* in a variety of ways.

The background spaces of written libraries are divided into books or internet pages, where the books/pages can be grouped by topics.

In the spoken language, the speech space is divided according to who is speaking to whom and when.

Then there are finer fragmentations within books, e.g. into chapters, sentences and, finally into words, which are marked by white spaces of variable size and by punctuation signs.

It may be, a priori, unclear if a given sign – white space, comma, capitalization, italizations, etc., are markings of the space (structure) or they are some kind of letters. Besides, some markers, e.g. the exclamation sign, play additional roles. We shall postpone this problem till ??? and assume we understand the "meaning" of these signs. (The "meaning" of an exclamation sign for us is just distinguishing the space it marks from that marked by the dot sign.)

Disclaimer. (Ergo)Linguistics, as much as (relativistic) cosmology, for instance, can not be "reduced" to any kind of known or even unknown mathematics be it the category theory or the *(pseudo)-Riemannian* geometry. However, one can not hope to invent a model of a non-trivial structure without playing with mathematical ideas around it, where the relevance/irrelevance of a particular idea becomes apparent (if at all) only after playing with it for quite a while. (Calculus will not bring you to the moon, but you can not even start planning your trip unless you understand calculus in depth.)

Also notice that the difficulty faced by an ergosystem (e.g. a child's ergobrain) in "undertsnding" a language is rather different from what is encountered by a linguist in *deciphering an ancient language*, where the meaning is less of an issue but the available supply of texts is limited. (See *http : //en.wikipedia.org/wiki/Decipherment* and references therein.)

The problem with our mathematical ideas is not that they are too abstract, too difficult or too farfetched, but that we lack imagination for pulling "abstract difficult and farfetched" ideas out of thin air. Nor do we have a foresight for predicting how an idea will develop.

We are confined to the circle of concepts, techniques and ideas of the present day mathematics; only abstract, general and/or universal among them, rather than those already adapted to solutions of specific problems, stand a chance for survival in a novel environment.

We play by modifying these ideas, but not by specialization, but by making them to grow into something even more "abstract and general" which, hopefully, would allow us to express the properties of the structure at hand in mathematical terms. To achieve this, we need to make a step backward, to strip off the closes of mathematical precision from the original ides, since we do not know what their final shapes are going to be; by necessity, our mathematics can not be too rigorous.

After all these excuses let us formulate the following

Vague Conjecture.

1. Every "real life", a priori, *unlistable* library $\mathcal{L}$ (e.g. representing a human language) can be "naturally" (functorially?) compressed to a *listable* dictionary $\mathcal{D}$.

2. Such compression can be achieved by some *universal* rules/tasks applied to $\mathcal{L}$.

Let us give a (very) rough idea of a "compression" we have in mind.

The structure of a library $\mathcal{L}$ resides in correlations of distributions of structural patterns in it. A *(ergo)dictionary* is a structural organization of these patterns and of their correlations, which makes a substantial part of (the memory of) a program Pro which "understands" $\mathcal{L}$.

The most primitive way to make a dictionary is by "folding/compactifying" a library into a metric space X as follows.

The points x in $X = X_l$ are strings of letters from the library $\mathcal{L}$ of certain length l, say $x = (a_1, a_2, ..., a_l)$, where a realistic l is somewhere between 10 and 30, the number of letters encoding a few words utterences

The distance between $x = (a_i)$ and $y = (b_i)$ is designed depending on the maximal $k = k(x,y)$, such that the initial k segments of the two strings are equal; $a_i = b_i$ for $i = 1, 2, ..., k$. (Geometrically, it is better to use central k-subsegments.) Namely,

$$dist(x,y) = \varepsilon^k \text{ for some } 0 < \varepsilon < 1, \text{ e.g. } \varepsilon = \frac{1}{2}.$$

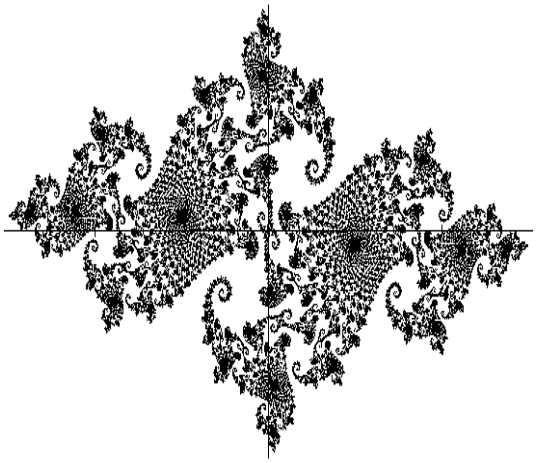
(Here, we ignore possible problems arising from white spaces and punctuation signs.)

This space X seems at the first sight as large as the library $\mathcal{L}$ itself, since it contains as many points as there are letters in $\mathcal{L}$, say $N \approx 10^{10}$: If you use short strings, than many of them become identified, but almost every 30-long string would appear only once in the library, if at all, assuming there is no repetitions of texts. (Notice, however, that this compression includes the factorization by the permutation group on the set of letters.)

The essential difference between X and $\mathcal{L}$ is the concept of locality: if you are positioned at a string x_0 in a text, what you see around you are nearby strings, e.g. the immediate continuation of x_0. But if you are in X the close strings are those which have large overlaps with x_0.

(Metaphorically, passing from a text in a library to a dictionary is like folding a polypeptide chains C to a protein $P = P(C)$. A chain has no physiological meaning/function in the cell. Folding erases the redundancies in the chain and give it a functional meaning(s) which is partially encoded by the position of P in the protein interaction network. Also, folding replaces the translational backbone symmetry of C by a folded architecture of P with less apparent structural symmetries.)

If we look at X with a limited magnification, we shall see it as a continuous entity, something like the above??? fractal image below???. Furthermore, if the points of X are made slightly luminescent, we shall see some distribution of light on X – a probability measure on X encoding relative frequencies of strings in $\mathcal{L}$. An informative characteristic of an x_0 in X associated to the metric and this measure is represented by the ratios of measures of δ-balls around x_0 of various radii δ.



Another way to "look" at a metric space X at a δ-scale is by taking a *δ-dense net* X_δ in it, i.e. a finite subset, where "δ-dense" signifies that every point x of X lies within distance $\leq \delta$ from X_δ, i.e. there exists a point x' in X_δ such that $dist(x, x') \leq \delta$.

The minimal number $N = N(\delta)$ of points in X_δ needed for δ-density characterizes the "size of X on the δ-scale". If δ is not very small then this N, hopefully, will be not very large – a moderately listable X_δ, say, of cardinality 10^5-10^8, will suffice for the δ-density.

On the other hand, the geometry of a δ-net X_δ, with the metric coming from the ambient space X, may give a fair representation of all of X. Moreover, one does not usually have to keep track of all $N^2/2$ distances between the points in X_δ, but only between the pairs of points within distance close to δ, say, $\leq 4\delta$ between them, where the number of such pairs is of order N, rather than N^2.

The space X, besides carrying a metric, has an additional structure, call it *$\mathbb{Z}$-foliation* where $\mathbb{Z}$ – the infinite string of integers – reperesents the backbone of the library. This foliation structure is just a recording of the neighborhood relation of strings in the library: $x' = (a'_i)$ is called the (right) *neighbor of* $x = (a_i)$ if $a'_i = a_{i-1}$ for $i = 1, 2, ..., l-1$.

The above fractal image is of similar origin: it is the trace of an orbit of a certain map, where the $\mathbb{Z}$ structure is given by (partial) orbits of this map. The string foliation of X is also generated by a map, where every string goes to its right neighbor, but this map may be multivalued and only partially defined. In both cases, the foliated (orbit) structure is geometrically invisible.

A similar "folding" is possible (and, actually, more instructive) for libraries of, say, two dimensional images, where X is made of small patches of such images with a suitable distance between them and where the corresponding foliated structure is based on (the symmetries of) $\mathbb{R}^2$ rather then on $\mathbb{Z}$.

Our space $X = X(\mathcal{L})$ does not look so beautifully symmetric as fractals depicting orbits of rational maps. (Of course, nobody has seen X anyway, visuslisation of it is a non-trivial problem.) Besides a true dictionary, say the one in your ergobrain, carries more structures than just metric and $\mathbb{Z}$. Moreover, some of the structure in the ergobrain language dictionary comes from the associated "visual dictionary". On the other hand, even this simple minded X

raises interesting questions. For example,

Is there a natural clusterization of X?

How much does $X(\mathcal{L})$ change (with respect to some natural metric in the category of *metric+measure spaces* [45]) when you change the library?

Do different languages have significantly different spaces X?

Are there natural geometrically robust invariants distinguishing these X?

Does a close relationship between languages make their X look similar?

Ideal Learner's Dictionary. Such a dictionary ID, should represent, for some δ, an optimal δ-dense set, in the "folded dictionary" i.e. the one with minimal $N = N(\delta) = N(\delta, \mathcal{L})$ and should be be suitable for an "ergo-learner" – an ergosystem or an intelligent alien from another Universe.

An ID would consists of a collection of N relatively short strings, say, up to 10 words, including lists of individual words, letters, and punctuation signs, where a realistic N for a human language may be 10^5-10^8 and, probably, somewhat greater for a "dictionary" of visual images.

The essential point is that similar strings are grouped together. Thus the spacial organization of strings in ID (understood by our ergo-learner) would correspond to their distances in the "folded dictionary", but now with a more "meaningful similarity/distance" than the one defined above which the maximal lengths of common substrings.

Moreover, an ID may be compiled of several volumes consequently having larger and larger vocabularies and longer strings, thus, making finer and finer nets.

None of this, however, allows an adequate representation of the "folded geometry"; no matter what you do, the ordinary dictionaries remain bound to the linear order on written signs, while the "true ergo-geometry" of ID is by no means linear.

(The nearest approximation to ID, I am aware of, is *The Advanced Learner's Dictionary of Current English by A.S. Hornby, E.V. Gatenby, H. Wakefield.* Its second edition, Oxford 1963, contains $\approx 2\cdot 10^4$ word-entries and $\approx 1.5\cdot 10^5$ sample phrases selected with unprecedented perspicacity.)

An ID may also adopt some common features of ordinary dictionaries, such as a use of different fonts/colors: for distinguishing *linguistic patterns* , some (not all) of which make syntactic and semantic *units of a language.* such as morphemes, words, phrases (but also including something like *man–him* pairs). Also, an ID may use (a rather short list of) symbolic *classification and disambiguation tags* which are collected in ordinary dictionaries under the heading of "Abbreviations", such as **n**. for "noun", **v**. for "verb", **iron**. for "ironic". etc.

Definition/explanations following the word-entries would be most useful for an ergo-learner if they "reduce" a δ-net ID to a *coarser* δ'-net with $\delta' > \delta$. This is not a common practice in most dictionaries where "cat" may be defined as "a carnivorous feline mammal...". Similarly, the definition of *noun* as

"any member of a class of words that typically can be combined with determiners..."

can hardly be accepted even by a developed human ergobrain, such as of an adult Pirãha native speaker, for example. But what will be helpful to an ergo-learner, and this how we want to design ergo-dictionaries, is giving a lists of sample words and indicating those you tag with **n**.. The ergobrain of any speaker of a language having nouns in it, will have no difficulty in extending the **n.**-tags to other nouns in the language, possibly, with many surprises for a pedantic grammarian.

Self-referentiality, such as in $noun \circlearrowleft_{\mathbf{n.}}$ is rather technical but expressions like "It has been already said that..." (abhorred by logicians) are ubiquitous in all (?) human languages. The presence of self-referential entries in a dictionary is an indicator of this being a *language* dictionary rather than an "ergo-dictionary" of chess, for example. Also, "understanding" self-referential phrases marks a certain developmental level of an ergosystem. (A computer program, which can not correctly answer the question "Have we already spoken about it?", fails Turing test.)

Making a dictionary consists, generally speaking, in performing three interlinked tasks, where the third one is essential for human languages.

PAT. *(Generalized) Annotation &Parsing*: Identifying, listing and structuralizing "significant/persistent **pat**terns/fragments" in short strings (say, up to 100-200 letter-signs) in a given library.

SIM. *Classification/Factorization*: Identifying, listing and structuralizing **sim**ilarities between "significant patterns" in strings which may appear far away in the library.

REF. *Implementation of Self-**ref**erentiality*: Applying PAT&SIM not only to libraries, but also to dictionaries as well as to suitably represented PAT and SIM themselves.

On Annotations and Tags. The task PAT can be seen as *annotating* texts $\mathcal{T}$ – something opposite to factorization/"folding" corresponding to SIM. An annotation consists of several texts written in parallel with $\mathcal{T}$ on a combinatorial graph-like structure $\tilde{B}$ extending the background (e.g. sequential, piecewise $\mathbb{Z}$) structure B of $\mathcal{T}$.

This is similar to parsing in computational linguistic – identification of patterns of "functionally related" words and *tagging/naming* such patterns (as well as words themselves), where the tags (names) make a part of the dictionary. However, unlike ordinary parsing, our annotation rules will not rely on any kind of a priori known grammar of the language/library $\mathcal{L}$.

An ergo-dictionary is obtained by a factorization/"folding" applied via SIM to a library of annotated texts. The metric (or rather something more general than a metric) on the "folded dictionary" is designed on the basis of similarities between annotated strings, with the $\mathbb{Z}$-foliation replaced by a kind of $\tilde{B}$-foliation. The (process of) folding/factorization may be (partly) encoded by *classification tags* assigned to (similarity classes of) linguistic units/patterns. (It is often hard to draw the demarcation line between annotation and classification tags.)

This may seem circular, since tags appear before and after folding, but this circularity is actually essential for building of a dictionary and/or evaluating its quality.

But granted a *complete* structural representation of *all* redundancies in a written language library (where "complete" and "all" are unrealistic), can we claim we understand what is written in there without extra annotations associated with the non-linguistic signal inputs, e.g. coming through visual and/or somatosensory systems?

Superficially, the answer is "No". If you are deprived of vision and somatosensory perceptions there is no signals to find redundancies in, there is no basis for understanding. Moreover, there is no apparent translation from linguistic to sensual libraries. No verbal explanation will make you able to drive a bicycle, for instance, or to perceive music.

On the other hand we learn in science to mentally manipulate "objects" which are far removed from what is perceived by our five/ten senses. (But as for bicycling, no verbal instruction will make you understand abstract science or mathematics, unless you start "mentally driving" it yourself. I guess, you can not learn much of an experimental science either by just reading, listening and talking)

The problem with non-lingustic inputs is not so severe as it may seem – this is the matter of *knowledge*, rather than of understanding, i.e. having an access to arrays of signals which come in the time flow and; thus are *listable*. But their redundancies, if straightforwardly expressed, say, by correlation between triples of signals are, a priori, unlistable: N^3 is beyond reach even for modest $N \approx 10^6$.

The essential problem is to outline designs of dictionaries *structurally* enlisting the redundancies in flows of signals and then produce an *universal* learning system for actual building such dictionaries. Of course, a structuralization of redundancies is possible only for rather special arrays of signals. But since our brain is able to do it, the signals *must* be special; mathematicians should be able to repeat what the brain does.

The language is technically most approachable for this purpose, since we do have digital libraries of languages at our disposal, unlike, say, somatosensory libraries. Besides, certain "universals" in human languages e.g. the concepts: "it", "and" , "then", "question/answer", "rarely/often" are suggestive of how the ergobrain structures "information".

On the other hand the natural languages, which have to fit into the one dimensional procrustean bed of *strings*, are the most difficult to analyze, since their "folds" within strings hide many invisible not-quite-local correlations. (When unfolded by annotations, the correlations become essentially local and rather transparent. Then one can fold libraries into dictionaries by factoring out their redundancies.)

A universal learning system/program which could build linguistic dictionaries, would have little difficulty, I believe, with other kinds of signals delivered to us by our senses.

A somewhat opposite problem, specific for languages, is that of a (partial) reconstruction of a human "ergo" (who cares about somebody's else ego) from a written text. Can one take literally what Horace sais: "...non omnis moriar multaque pars mei uitabit Libitinam..."– (not of all me will die, many bits of me will evade burial)?

This hardly can be answered at the present point.

4.3 Short Range Correlations, Segments and Words.

Our present models of ergo-annotations and of ergo-dictionaries will be based on combinatorics: a dictionary for us is a *listable* set of certain "units" which represent "ergo-ideas" and where these units are organized according to some relations between them.

The basic units of a human language are *words*. What are they? Can you explain the following definition to a computer program?

"A word is the smallest free form – an item that may be uttered in isolation with semantic or pragmatic content".

Instead of stepping into the "definition game", let us enumerate the common properties of the words or rather the *properties of languages*, which allow (almost) non-ambigues *segmentation* of (long) strings of letters into (relatively) short ones – "words".

Children learn in school that

(A) *A word is a string of letters in a language which admits no insertion of another word between the letters of this string* – if you insert a word inside a word, the resulting string will be *not* a string in the language. Also you have to rule out *parts* of words which do not make words themselves. (This is taken for granted by children, but it must be made explicit in our case.)

This corresponds to the idea of a "physical object" as a maximal *connected* shape in space. ("Maximal" is needed to rule out all conceivable parts of objects which do not make objects themselves.) The commonly observed "connectivity" is due to the fact that the physical interactions are short ranged on the perceptible scale:

if two object are separated by "emptiness", they eventually go far apart in accordance with the Second Law of statistical thermodynamics.

Long range interactions in physics (and separable affixes in German) smell of magic, our mind refuses to accept them. But once accepted, groups of several interacting objects make new objects.

For-instance the *solar-system* makes a distinct word (world?) in *obedience-to* the *inverse-square-law* , where the *planetary-orbits* of the six (including Earth) *heavenly-bodies nearest-to* the sun are positioned, *according-to* Kepler, in *concordance-with* the *geometric-invariants* of the five *Platonic-solids.*

(The boundaries of words in most alphabetic writing systems are *conventionally* marked by white spaces. These conventions greatly contribute to the arsenal of ingenious spelling devices employed in schools for torturing children in civilized societies.)

(B) *A minimal (non-empty) string of letters pinched between two words makes a word.*

Thus, a short dictionary D allows one to identify most (all?) other words by looking at the segments between those words (in *sufficiently many* texts) which were already listed in D and where this process accelerates as D is augmented by newly discovered words.

Apparently, this is because languages were "designed" for communication. Empty spaces between physical objects do not make objects, neither do intergenetic and inter-*exon* segments on (eukaryotic) DNA look anything like genes or exons.

(Messenger RNA of many proteins in *eukaryotes*, e.g. in plants, worms and people, are composed of several segments which are transcribed from *disjoint* segments of DNA. These are called exons; they are separated by *introns* – non-transcribed segments of DNA. A random perturbations of an intron sequence rarely has a significant physiological effect on the organism, whereas such a perturbations of an exon may be deadly.)

(C) Statistically speaking, words (as wells as morphemes and shorts phrases) are exceptionally *persistent* strings of letters with anomalously *strongly corre-*

lated substrings in them, where the boundaries/junctions between consecutive words are characterized by *jumps* of the correlation function, which facilitate word prediction by an ergo-learner.

Another way to put it is that a segmentation into words (along with indication of morphemes within words and of common short phrases combined of words) and a (relatively short) word dictionary provide an effective structural formate for encoding the redundancies in texts which originate from short range interactions/correlations between letters (where this "short range redundancy" can be numerically expressed in terms of Shannon's entropy).

Let us give a rough idea of how this can be formalized.

Alphabets, Texts and Strings. The alphabet is denoted by A and its cardinality by $|A|$.

Texts are denoted by $\mathcal{T}$, where the number of letters in $\mathcal{T}$ is denoted by $|\mathcal{T}| = |\mathcal{T}|_A$.

Strings of letters in a text are denoted by s and their lengths by $|s|$.

$[s]$-*Strings* are strings of letters taken out of context and denoted by $[s]$.

To keep a perspective on *multiplicities*, i.e numbers of copies of strings $[s]$ in texts, imagine that the alphabet A consists of $32 \approx \sqrt{1000}$ letters with no specification of white spaces and punctuation signs (which may or may be not included into A) and that the letters appear in texts with comparable frequencies.

Then the *mean gap*, i.e. the expected interval, between two appearances of every letter $[a]$ in a text, is about 32, and, in the absence of correlations, the mean gaps between copies of a short string are

$$\mathrm{gap}[a_1a_2] \approx 32^2 \approx 10^3,\ \mathrm{gap}[a_1a_2a_3] \approx 32^3 \approx 3\times10^4,\ \mathrm{gap}[a_1a_2a_3a_4] \approx 32^4 \approx 10^6.$$

where, observe, a printed book page contains 2000-3000 letters on it. Thus, a *random* 4-letter string may appear *at most two-three times* in a majority of 400 page books with $\approx 10^6$ letters in them.

When it comes to longer strings, counting becomes a suspect. Indeed, a "library" of all 6-string would contain $\approx$1000 books, about as much as what you read in the course of your life

Only about 1% of the 6-letter strings $[s]$ in 32 letters may come up in a 10^7-text ($\approx$ 10 books) since $32^6/10^7 \approx 100$; moreover only exceptional few among these $[s]$ (if any) are expected to have multiplicities≥ 5.

In reality, a predominant majority of strings of length six (marked with white spaces, to be sure) do not appear at all on any page of any book in the world although $32^6 \approx 10^9$ corresponds only to thousand books, since the words of length 5-6 in natural languages are relatively few, 10^4-$10^5 << 10^7$ of them; by necessity, many of these have multiplicities ≥ 25 in most 10^7-letter-texts with

"before", "change", "should", "answer", "always", "second", "enough",

being plentiful in almost every book.

If you pass to, say, 9-strings, their number jumps up to 30 trillion – about 30 million books.

For comparison, the Library of Congress contains about thirty million books, but I guess, there are many copies of the identical texts in there. (A likely bound on the total number of *different* books, understood as 10^6-strings, in all languages in the world is $\approx$50 million with about 10 million authors.)

If this library were "random", most strings, say of length 11-12 would appear only once (since 32^{12} letters make $\approx 10^{12} >> 3\times 10^7$ books) but this is certainly not what happens.

If you come across a twelve letter strings $[s]$ on a random page of a random book in a (even moderately) large English library, you expect that this $[s]$ will come up many times again and again. (This would be even more so for strings s pinched between white spaces.)

In conclusion, it is unreasonable to regard the set of "words" of length l in a natural language, say starting from $l = 5$ or 6 as a subset of all strings of length l (or as a probability measures on the sets of *all* strings). Only "unreasonably *persistent*" strings stand a chance of passing the "word"-tests.

How to measure "persistence" of a string $[s]$? Its multiplicity $m_{\mathcal{T}}[s]$ in a long text $\mathcal{T}$ is deceptive, since a short $[s]$ may appear *only* as a part of a longer string $[t]$ or of relatively few different $[t]$. For example, certain books come up in tens of million of copies. With this in mind, we say that

$$[s] \textit{ comes up } m \textit{ times in a text } \mathcal{T}$$

if there are m *different* strings $[s'ss'']$ in $\mathcal{T}$, where s' and s'' has certain not very large length l say $l = 20$, where, observe, this $m = m_l[s]$ called *multiplicity* of $[s]$ is essentially independent of $l \geq L \approx 10 - 15$.

This multiplicity itself is too much text dependent, but the variation of m, as we adjoin a letter to a string or erase a letter, is more (but not fully) meaningful, where, moreover, the value of the *second* variation ("multiplicative Laplacian" $\Delta^\times$) is indicative of a word boundary.

Indeed, write $[s] = [s'a_1a_2]$ and $[s] = a_{-2}a_{-1}s''$ and let

$$\Delta^\times[s..] = \frac{m[s']/m[s'a_1]}{m[s'a_1]/m[s]} = \frac{m[s']m[s]}{m[s'a_1]^2},$$

$$\Delta^\times[..s] = \frac{m[s]/m[a_{-1}s'']}{m[a_{-1}s'']/m[s'']} = \frac{m[s]m[s'']}{m[a_{-1}s'']^2}.$$

Naively, if a string $s'a_1$ makes a (typical longish) word then s'_1 *uniquely* extends to $s'a_1$ and $m[s']/m[s'a_1] = 1$. But adding a letter a_2 which starts another word makes $m[s = s'a_1a_2]$ significantly smaller than $m[s'a_1]$; thus, $m[s']/m[s'a_1] << m[s'a_1]/m[s]$. This means that the predictive power of an ergo-learner jumps at the word junction (as it happens to a bug at the edge of a leaf).

More generally, if

$$\Delta^\times[s..] << 1, \text{ i.e. if } m[s']/m[s'a_1] << m[s'a_1]/m[s],$$

then a_1 is likely to be the right word boundary letter, and similarly $\Delta^\times[..s] << 1$ makes a_{-1} a good candidate for the left word boundary, which means in both cases that the m-function is expected to be pronouncedly concave (rather, *log-concave*) at the ends of the words.

This can be reinforced by using several strings, with the same right end letters as $s'a_1$ (say, with 3-5 such letters) and, similarly, for the strings starting with the same letters as $a_{-1}s''$.

The above is meant only to indicate how (our very coarse version of) the usual statistical approach to the word segmentation (see ???? ???) implements the prediction maximization drive by an ergosystem; such approach can be used for segmentation of visual images (where an essential new problem is deciding which patterns are regarded as "equal") but it has limitations as we turn to the true languages for the following reasons.

1. Letters are artifacts of writing systems. Segmentation into words must be accompanied by identification of morphemes on the one hand and of short phrases on the other hand. (If you represent letters by binary numbers, then you see clearly what may go wrong with ± 1-letter variations in the extreme case of a language with $|A| = 2$.)

2. The $\Delta^{\times}$-based segmentation (and its refinements) are based on the assumption of absence of long range (say more than 5-6 letters) correlations in strings, which is not the case in human languages, where the syntax (and semantics) is not that local

3. An ergo-learner has no idea what a string is and he/she does not bother counting letters in texts; his/her memory retains only relatively short strings which came up sufficiently many times within not very long time intervals and/or are significants in other respects. We shall see in ??? that this may work better than the traditional statistical analysis of long texts.

4. The words in text marked by white spaces are, up to some extent, conventional and you do not expect an ergo-learner to guess what this convention is in every particular case. Eventually, an ergosystem my arrive at a segmentation different from the conventional one.

On the other hand, the presence of a "letter" designating the white space, even if an ergo-learner does not know its meaning, makes the word segmentation an easy task to an ergosystem. Since this "letter", let its usage be sometimes purely accidental, is present in the actual texts, there is no need to spend an extra effort for identifying the word boundaries.

Segmentation of texts into words makes a small (but inseparable) part of a broader and more interesting process of structuralization of texts; the second essential ingredient of this process is described in section.??? aaa

5 Classification of Words, Tags and Inter-word Correlations.

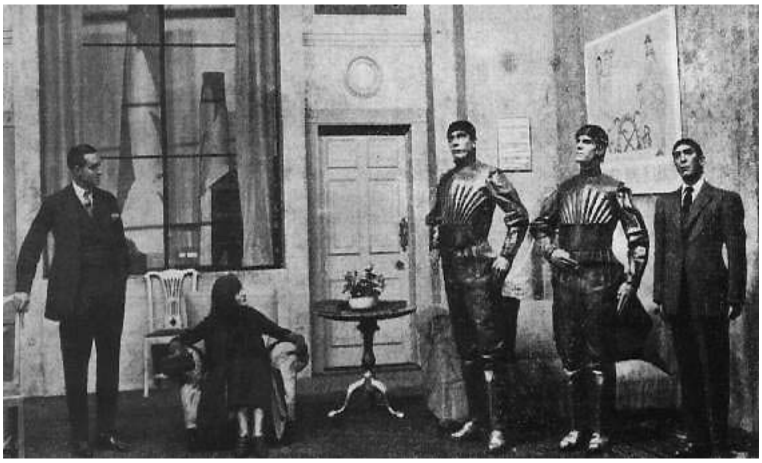
6 Mundane Stories.

The last person you would suspect of being a humorist, Ludwig Wittgenstein, writes:

" ... philosophical work could be written consisting entirely of jokes."

Being a cursory reader of philosophy, I could recognize humor only in Voltaire's Philosophical Stories and in Terry Pratchett's Discworld series that are candidly made of jokes. Apart from sarcastic maxims by Nietzsche, I naively believed that all philosophers were dead serious and literally meant every word that they were uttering. But reading this sentence opened my eyes. For centuries, philosophers were poking fun at unsophisticated readers, especially at scientists and mathematicians, who were taking everything they read for real.

Thus, enlightened, I attempt to make the subtle humor hidden in some treatises on philosophy of science, mathematics, language and artificial intelligence



accessible even to a most simple-minded mathematician.

6.1 Self Referential Loop.

A Turing machine can be always confronted with a question which it either answers incorrectly or it steps into a self referential loop and will run for ever without stopping. This, of course, only applies to Turing robots which were programmed to obey, which is not the case with ergo-systems, which, albeit non-adequately, can be modeled by Turing machines. But if making a full-fledged ergosystem is a non-trivial task, safeguarding a robot from awkward questions is an easy matter, where, moreover, one can safeguard the human ego from being hurt at the same time. There are several programs on the market for protecting human-form robot's delicate core software from such loops; yet, even the best of them are not free of bugs.

We walked with Arnie through the city park talking about robots.

Arnie held a doctorate in zoo-psychology from Harvard. He published a year ago a controversial article in Nature: "What is time to a rat?", where he argued that rodents, unlike primates, had a milti-linear time perception.

He currently worked for the "Robot Beautiful" an expanding robot renting company. The executives in RB figured out that their robots could learn something from animals, especially from dogs – friendly, happy and only eager to serve.

Arnie did not give a damn for RB. Arnie loved animals. He called the RB robots "painted sissies" – as intelligent as rubber policemen – simple handheld devices used in the chemistry labs. Yet, he kept with RB for almost four years.

The RB "sissies" revolutionized TV commercials. The robots radiated trust and confidence: homely features, irresistible smiles, perfectly adjusted voices. No human could look so honest and natural. The sales of the robot advertised products boomed.

RB were also leasing the robots to the city public departments and to busy rail and bus stations. The robots had millions of ready-to-use sentences in their fast memories and could pick up phrases from the internet. They directed peoples to taxi stands and to WC's with encouraging smiles. They selected

jokes with 90% accuracy.

The success of RB, Arnie was explaining to me as we walked, was due to the artful work of the hairdressers employed by RB, least of all to a creativity of the RB robot designers.

The “listen and talk” program, as Arnie explained to me, had been dreamed of by his cousin, Richard Gross, an eccentric neurophysiologist. The core software was compiled by a team of bright computer programmers under Richard.

– The ”designers” – Arnie was saying – feed vocabularies into robots. This is all they know. Not the software. They understand as little in it as mathematics professors in the psychology of their students. They are OK to correct robots’ grammar. But when it comes to ”loops” they run to Richard’s boys for help.

– If something goes wrong a robot steps into a loop – it freezes. This happens much too often – sissies are touchy when it comes to confronting their own selves. This disequilibrates them. We, primates, are also like that. A male capuchin monkey goes hysterical when he tries to negotiate “who is alpha” with his mirror image.

– The freeze protects the software and it is better than hysteria – who wants hysterical robots around. This what Richard says.

– But, knowing Richard, – Arnie continued – the “loops” are there for the sake of ”designers”, to make fools of them, to keep them on their toes, so to speak.

–You have to see it – He chuckled – a sissy stepping into a loop with an idiot smile on its face. So confusing!

– Confusing?

–It is the “loop emergency string”— three simple words, “letting off the steam”, as Richard says. The words come just before the freeze. They go, these words, with no articulation of the mouth and lips. A robot who is about to freeze kind-of ”throws” its voice, as ventriloquists do. The words seem to come from whoever triggered the loop. This makes it ... confusing.

Arni's eyes sparkled with hidden laughter.

I felt curious– What are the words?

– We never say these words aloud, robots may pick them up and install into their working memories. You know, they can learn new words from people. If the words come at a wrong moment this may be ... confusing.

– But there are no robots here in the park.

– Who knows. Some Helen's or Jenkins' robots may be around. The city safety department planted some at public places. They think it scares off drug pushers. The robots have pretty good hearing. They may recognize my voice, take it for a teaching session, record the words and say them in my voice somewhere else. This may be.... confusing. – He chuckled again.

Helen and Jenkins were Arnie's coworkers, "designers of robots". Helen– Arnie complained– was "humanizing" her robots by feeding "personal memories" into them.

– If, accidentally, you touch this memory, her sissy starts mumbling: "Yes, Arnie, this is a very good point. When we with my younger sister Helen, while playing hide-and-seek at the 2007 Christmas party..." Only mini-Turing can stop that.

– Jenkins is another kind of character. A robot of his may go around reciting: "Mr. Jenkins is my creator. He conceived me in his head. He shaped me with his bare hands. He installed the software into me. He...". Tell the painted piece of rubber what you think of its "creator"' – and the string is on you.

I felt Arnie was trying to distract me from the magical words.

– Hey! – he exclaimed suddenly— look, see a robot over there. He pointed to a bench where I saw broad shoulders of a man in a T-shirt and a red haired head over a laptop.

– If you want, Ryan, I will make him say the words.

I did not like the mischievous sparkles in his eyes, I knew Arny too well by now.

– But wouldn't it disable him, Arnie?

– Do not worry, I know how to fix him back.

– Are you sure this is a robot? He does not look a sissy.

– I am a field biologist – Arnie sounded hurt –I can tell one black cat from another in the dark by the tail. I know this copper red, the pride of our hair artists. No human can have such metallic iridescence in the hair. And if he is a man, after all, what I am going to say only will make him laugh.

– What is it going to be?– I knew robots were harmless but to deliberately confuse a robot...

– Well – Arnie was hesitating – I have to mix Helen's and Jenkins words to make sure.... Something with "younger sister Helen"... "conceived by creator".... "bare hands" ... . OK, I know what to say.

This happened too fast...

We stopped eventually catching our breaths. Fortunately, the copperhead decided not to run after us, but we felt safer to be out of his sight, in case he had changed his mind.

Arnie said proudly – It worked, you heard the words.

– You mean...– Yes, the first string of three words.

– The words were quite expressive, I admit, but why these? They are not for sissies.

– Richard picked them up from an old robot movie – not at all for sissies:

– Two hundred pounds of muscles is repairing his eyeball with a screwdriver in a shabby hotel room between fights with another robot.

– The desk clerk knocks at the door.

– This annoys the robot, he searches his memory bank and comes up with this string.

– The clerk hurries away – the words were convincing.

–"Convincing"... I take it. But why this redhead was so dynamic? Frozen sissy? Frozen or not, robots can not improvise. He must have searched internet with lightening speed, judging by how eloquently he kept "convincing" us as we were running away from him. I've never head anything like this.

– Strange...– Arnie reluctantly agreed.

– Arnie, this robot did not look frozen to me.

– Yes, he *was* frozen, just for a fraction of a second after he heard my question, you had to see the expression on his face – no smile there.... *Then* he unfreezed, emitted the emergency string and turned hysterical. The movements of his jaw *were* correlated with the articulation of the words all along... Impossible!

Arnie closed his eyes concentrating.

– Oh! How could I forget?!—Arnie relaxed and continued lightly— They started advertising this copper hair dye on TV. Must be sold everywhere by now. Why on earth people want to look so much like robots?

– But if he was a man, why didn't he laugh? What was it you asked him, Arnie, to make him so mad at you? Be he man or robot his language was rather excessive.

Arnie smiled – I do not remember how I phrased the question.

He paused and added apologetically

– I got so much used to dealing with robots, Ryan. Men psychology is so confusing.

– Don't you find, Arnie, his vocabulary was rather unusual? Most of what followed the "string" I had never heard in my life. And what a voice! You do not have to be a drug pusher to be scared away. And then there was something about robots, the last what I heard, when we already were far from him.

–True, it now comes back to me... Strange... this vocabulary. And coming to think again – his hair. I could not be mistaken that much. I recall, they even complained about this dye, it did not look quite as good as the real robot hair tissue... Another prank by Richard maybe? A hysterical robot? I will ask him tomorrow.

6.2 Turing Test.

Can a computer, a machine, an automaton, a robot be taken for an intelligent human being? This problem goes long back in history. A currently accepted detection protocol was suggested by Turing in 1950. It was a subject to many objections, refinements and modifications by several generations of science fiction writes, philosophies and computer scientists. What is the magical something *inside* our heads which distinguishes *real* people from human-form robots? If you meet a complete stranger on the street can you tell if this is a human being?

Till recently this was a purely academic question, but now-a-days you have to look for all possible clues.

Jason touched his thick head of red hair with uncertainty. He used to be proud of his hair. Only Flynn's in the city of Galway had such magnificent red in the hair. This was a mark of distinction. And now... the red was spreading like fire. Jason was contemplating of shaving his head.

He was on the plane returning to New York after a brief visit to his parents. New York – the epicenter of this fire! And this young woman across the aisle with a mini-Mac. Jason was detecting her barely concealed glances. He was not especially shy with women and would not have minded if she had stared at *him*, but she looked only at his hair – nothing playful – only a dry curiosity.

He turned away from her, certain of her gaze. Then he turned back, very fast, unexpectedly and he caught her eyes.

Now she stared at him, at his face – no embarrassment, a shear surprise.

– You can't be a robot!– she exclaimed.

– What! – Jason's features froze in incomprehension, his mouth open.

– Not a robot – she repeated, blushed and turned away.

– Robot? Why robot?

She turned back to him.

– I apologize, this was stupid of me, you must have your reasons. I do not want to pry, It is not my business, really – and she turned away from him again, unwilling to continue the conversation.

Jason was taken aback. He saw she did not want to talk but could not resist. He inquired with a touch of sarcasm in his voice.

– Hm, I am not a robot. But can you tell me what it's all about?

She answered without turning away from the screen of her computer.

– Isn't it enough? I have apologized, let us not play games. You can not be that naive.

– Games!? Naive?

She look at him again and half-smiled.

– If you *are* naive, you are playing a more dangerous game than you realize.

– Look, Lady. First you take me for a robot because of my hair, I guess. But you see thousands of such redheads since they started advertising this hair dye. Now you tell me I am not a robot and that I am in danger.

– Stop pretending, you were quite deliberate letting me know you were not a robot. As for a danger... this may be an exaggeration, but it may turn awkward if you are taken for a fast robot. Now, after I've answerd your questions, will you satisfy *my* curiousity. I did not want to pry, but you do not seem to mind. Why?–

She touched her hair.

– It must've been quite painful.

Jason's mouth opened again.

She sighed with resignation.

– I am in no mood for games.

And she turned back to the screen of her Mac.

– Please, enlighten me – He pleaded, you have not answered anything. What is painful about my hair?

– Do not tell me this is a wig – she responded with irritation. I looked at your scalp carefully.

– Of course, this is not a wig, it is *my* hair!

– And next you will tell me you have grown natural antlers and a peacock tail. Enough is enough!

Jason felt exasperated.

– Listen, I've lived with this hair for thirty years, how can I prove it to you!

She smiled impishly bent down and took a micro-scanner from her bag.

– Do you want to see it on the screen, the matrix?

– What matrix, this is just hair, do you want a bet, you will see no matrix!

– Your bet is as good as lost, this is a custom made laser scanner, with more than ×300 resolution in the infrared. Ready?

– And if *you* lose?

– I will eat my Mac Apple.

– Bone appetit– and Jason bent his head toward her across the aisle.

– Watch the screen– she said –no, wait, you have lots of hair, let me bring it to the skin. Now, you see?

– See what? You better look yourself.

She looked at the screen, gasped, and looked at Jason almost with awe.

– Incredible! This is *not* an implant!

– Of course not – Jason said triumphantly.

– But how you've got this hair?

– I told you I was born with it, or they grew soon thereafter. Why do you doubt everything I say? Better explain to me all about hairs, robots, dangers. antlers, etc. Let the apple go.

– But will you promise me something?

– To tell the truth, the whole truth and nothing but the truth?

She blushed.

– No, – I assume all you said was the truth–

But she sounded doubtful.

– You will promise to give me several your hairs.

– So romantic

She blushed again,

– I want to look at them in my lab.

– Agreed, Now your story.

She took a deep breath.

– You see, your hair does not look human. Human hair, and the fur of most, probably all, I am not a zoologist, mammals do not have a pronounced periodic structure on 100-nano-scale. There are some quasi-periodic insertions of pigments here and there but these are not very regular. Quite different from plumage of birds and from insects for that matter. Understandably, I was confused.

Now, give me what you promised, I've explained everything.

– Wait, wait. All I've got so far is that I am not pronouncedly a beetle on the nanoscale.

– Do I have to spell it out? You are not a child.

– Assume I am.

She sighed again.

– OK, iridescence depends on light interference on regular patterns, periodic with the about green light wave length. Human red hair is, at best, only mildly iridescent, not like the peacock tail, for instance. There are artificial iridescent materials, I produced myself quite a few new ones, but it does not work so well with dyes, obviously. Your hair is abnormal in this respect. Clear now, isn't it?

– And what made you to change your mind about me?

– This is too much! You had shown this to me yourself, do not say this was accidental.

– Didn't we agree you would be patent with me as with as child?

– But would you insist that how you turned to me was by chance? Do you habitually turn your head with such speed?

– Not usually.

– So?

– God! Can you stop with the puzzles? What does my head have to do with robots, apart from the color?

She looked at him inquiringly and eventually smiled.

– You know nothing about the current robot laws in US, do you? Do you know which century we are living in?

– Which laws?

– Robots are not supposed to harm anybody.

– Have I harmed you in any way?

– You are impossible! Physically to harm somebody you have to move fast and with a sufficient force. The mechanical design of robots does not allow this. Besides, their batteries are limited to something about fifty watts, less than half of what you normally use.

– I am starting to understand. You say Robots can not move fast... . But I saw myself two running like hares.

– What?! Are you with the military?

– Me, why?

– I guess only military would dare to make such robots, if they could. Where did you see such robots?

– In Central Park, a couple of weeks ago.

– This is impossible!

– Now again, you do not believe me!

– OK, I believe, how did it happen?

– I was sitting on a bench composing a message to my sister. The moment I sent it, two robots approached me.

– I see. They took you for a drug pusher, may be. They are supposed to scare away drug pushers. Were you scared?

– Me?! *They* were scared, I told you, they were running like hell away from me.

– How did you scare them and why?

– I did not like what one of them said, this made me angry.

– Something abut drugs, police?

– I do not remember exactly, but no drugs or police were mentioned. He asked me something absurd, only a robot could make such a question.

– Something funny?

– Not at all, something quite unpleasant.

– What did you answer?

– I do not remember, I got very angry. This scared them away.

She gave him a sidelong glance.

– This I do believe, You may be scary when you get angry. But I do not think these were robots, just pranksters. Were they teenagers?

– Not at all, they were in their forties, I guess. Looked rather respectable.

– Still, people do silly things, unlike robots. What made you think they were robots.

– The question. At the first moment, I did not take them for robots, but then it dawned at me. If I knew they were robots I would not get so mad.

– But what was the question? Possibly they were tourists asking for directions and you misunderstood their English.

– They were, from Downtown, at least the one who spoke to me. And he had the "Hahvahd" touch to how he spoke.

– How could you tell?

– Accents is my hobby.

– Can you tell where mine comes from?

– It is obvious, you have stayed in Downtown for quite a while...

– Not much of a guess.

– Let me finish, you were born in England, you studied in Oxford, you spend several years in the south of France and you spoke some North Germanic in the family.... Icelandic, most likely.

– You are a dangerous man Mr. Higgins.

– Flynn, Jason Flynn, and not dangerous, most of the time.

He looked at the woman expectedly.

– Eva, my name is Eva Helguson. I still do not understand what kind of question it was, what made you *dangerously* angry.

– It was sort of personal.

– Personal? You knew the men?

– Never in my life, this was how I realized they were robots.

– Now *you* speak in puzzles.

– Well, it was something related to the message I was sending, no human could guess what was there but the robot did just that. They have a direct access to the internet, right?

– They can not read anybody's mail, no more than you can, unless specially programmed, which is illegal. Most likely you "robot" just touched something of your personal life by an accident, maybe he took you for somebody else.

– Out of the question, he had the right names etc.

Jason hesitated

– I'll tell how it was. It was about my younger sister Ellen. She had a painful brake with her boyfriend, certain Jenkins. If I meet him again... Anyway, the robot picked the names and everything from my message over the Internet and came up with this disgusting question. Something convoluted with "Ellen's shape" and "bare hands" of this rascal Jenkins in it.

– Indeed, this is bizarre.

– You do not believe me?

– I do.

But she did nod look convinced.

– Now, you've promised...

– How much you'd need?

– A couple will do.

– Take it, all yours, but I want to know the result. To be assured I am not a robot. Send it to jflynn on Gmail. With "y" and two "n".

She smiled, – I will do it. My word.

They were approaching the baggage carousel. A redheaded robot was explaining something to a woman with a small boy. A boy turned his attention to Jason.

– Ma, see, another robot!

Jason looked unhappily at Eva.

A stocky man looked at Jason then at the robot disapprovingly and murmured:

– Who made these things?

The robot turned to him and to Jason and said with a charming smile.

– Sir, I am happy to tell this to you. Mr. Jenkins is my creator. He conceived me in his head. He shaped me with his bare hands....

The stocky man interrupted this.

– I would brake these hands if I could!

The smile froze on robot's face. Then a three word profanity thundered through the hall. A total silence followed.

Eva giggled.

– A nice loop – she said.

The woman with a child pointed her finger at Jason.

– That's him! He killed the robot who was so nice to me. And such words... He scared my sun. Poor Jenny.

She hugged the boy closer to her and glanced at a meek man next to her with disapproval. The boy was looking at Jason with admiration.

The stocky man joined in.

– Robot killer! Call for police.

– Police! – somebody shouted.

Two security men materialized from nowhere.

– This man...! Robot...! Killed this robot...! – People were shouting.

The security men approached Jason.

– Sir, you disabled the airport robot, please, go with us.

Jason was about to open his mouth but Eva touched his sleeve. Jason went with the men.

– I go with you, I am a witness – Said Jenny's mother

– You – she turned to her husband – take Jenny home, and, remember, two large black suitcases, a small red one and two bags.

– I do not want home – Jenny whined

– The poor is scared, the woman said and looked at her husband as if it was he who had scared their sun.

– OK, darling, you stay with me– and – to her husband – Do not forget, count the luggage, six, rememeber.

They started walking, boy's eyes on Jason. Eva joined them.

– Are you realated, Madam?– A security man askeed her.

– I am a winess.

– *I* am the witness! – yelled Jenny's mother.

– You are not allowed with us, Madam – the security man said to Eva – unless you are a relative.

– I am – Eva hesitated – his comapanion.

OK then – the man shrugged.

They followed through the hall to a side room, one of the men carring the frozen robot efortlessly.

– They are not heavy – Eva said – catching the surprize in Jason's eyes – about thirty pounds, a plastic foam makes most of the volume.

– But now, do you believe my story? – Jason said

She sighed – You are a dangerous man to bet with. But please, let *me* do the talking, OK?

They enetered the room with several immobile robots attached to the wall – non-smiling, and a tired man at the desk with a half a dozen telephones on it in all colors. The man carring the smiling robot adjasted it to the wall and switched on the magnetic holder.

Jenny looked around apparently non-impressed with the robots and telephones. His eyes turned to Jason.

– He is the *real* robot. I will be a robot when I grow up. Big like this robot.

–You are not a robot, darling – his mother said with an angry glance toward Jason.

– I want to be a robot– Jenny insisted – *real* robot Then, glancing at Eva – I will have a companion.

Eva blushed, Jenny's mother look at her contemptuously.

The man at the desk turned to them – What happend?

– He killed the robot with dirty swear words – Jenny's mother said pointing to Jason – I am the witness. And he scared my sun. This is illegal.

– OK, Madam, will you make a written statement? Sit down, please. Here is the form.

The boy turned to Jason again.

– Show it to me, please. How you do it? You did not open your mouth. Mummy, why daddy never says it to you? Is Daddy a real robot?

The man at the desk asked Jason.

Can *you* explain what happened, Sir?

Eva interfered with a tone of natural authority in her voice.

– The robot has misfunctioned by itself, this happens to the R02 models. If you connect to the RB controller, it will be fixed in a minute time.

– But this was no reason for swearing in a public place – the man looked at Jason – One could hear it even in this room.

– I... – Jason started answering but Eva interupted him.

– Please, call RB, sir, and they will explain everything.

The man took the read phone and dialed.

– Please, Sir, – the boy pleaded Jason – do it again. Daddy never says it. Daddy is not a real robot.

The man eventully connected and was explaining over the phone.

– We have an accident with a robot here... –

– What its ESR number –

– Where...–

– On his chest, unbutton his shirt –

One of the security man did it.

– It is R-0204J11 –

– Right, I see the loop. It is frozen with a smile on its face, correct?–

– Yes. –

– How did it happen –.

– There was a loud swearing by an.... –

– Do not tell me what it was, I know this model –

The man on the other end of the line chuckled – It was loud, wasn't it?

– Very loud –

– Good.–

– Can you fix it?

– No, we can not do it, the protocol is quite complicated, only people from Dr. Gross' lab are qualified. I will leave a message with Dr. Gross' secretary and somebody will come and fix it to-morrow.

– Why not to-day? And what shall I do with another robot?–

– Another robot?– What is the problem ? What's its ESR number ? –.

– Jeff, check its number – The man pointed at Jason with the telephone receiver in his hand.

The security man moved toward Jason.

Jason turned red, hardly controlling his temper.

– Please, keep away from me –

– Don't be afraid boy, It will not hurt, I will just look at you number.

The man extended his hand toward a button at Jason's shirt....

The three word profanity thundered again, deafening in the small room.

The security man stopped, frozen in his step.

Jenny shouted happily

– He did it, he did it! He is the real robot. – and then, to Jason with reproach:

– You opened your mouth, it is not fair. Do it again. Do not open your mouth.

The smiling robot attached to the wall unexpectedly came to life and started chanting:

– He installed software into me. He gave me the power of reason. He...

– Will you shut up – Jason roared.

– Yes, Sir, I will shut up, Sir – The robot grew silent.

The voice from the receiver shouted,

– What happened!? I heard it! Now you have the second frozen robot. What is its number, tell it, tell it to me.

– We tried to get the number, you heard his reaction.

– Is it frozen, with a smile?

– No smile and very much unfrozen.

– Who said it was a robot?!

The man with the receiver in his hand looked around.

– Who said him was a robot? – he asked nobody in particular.

– I did – Jenny said – I will be a real robot. I will have red hair and companion.

– You see, how badly the boy is scared – the woman angerely said – Jenny, let's go.

– I do not want home. Please, say it again, as in the big room, do not open your mouth –

He looked pleadingly at Jason. Jason turned to Eva

– What does the boy want me to say?

– Just what you bellowed a minute ago. You didn't forget the words, did you?

– I... – Jason looked embarrassed – do not remember ...

– Let's go – the woman said – let's go home, Jenny.

The boy did not look happy to move, but then his face brightened.

– I will show it to daddy. Daddy will be real.

The woman gave Jason a glance of indignation and carried the boy away.

When Eva and Jason were leaving, nobody tried to stop them, they've learned the lesson.

– Was that how you scared the "robots" in the park? – She asked – But, of course, you do not remember. – She paused – I have an idea... I bet I know him!

– Know whom?

– Your "robot" in the park and I know who the *real* robot was.

– You can tell who is the real robot? How?

– It is easy – she smiled and glanced at his hair.

6.3 Bug in a Bug

An *avatar* of a gene is an entity controlled by this gene that maximizes the survival of the gene. The avatar of a deleterious gene is typically smaller than the physical body of an animal carrying this gene, e.g. that for a cancer-promoting gene in a cell in the body. On the other hand the bodies of the hosts of certain parasites serve as the avatars of parasite's genes that control the host morphology and/or behavior to their advantage. Examples are many.

1. A species of *tetradonematidae* infecting an ant makes ant's body to resemble a fruit that is consumed by a bird. The ants are infected by the nematode while foraging on bird's droppings.

2. Adult *gordian worms* reproduce in water. The juvenile worm parasitizes grasshopers, locusts and beetles and induces them to commit suicide by drowning themselves.

3. A *Glyptapanteles wasp* lays its eggs in a baby caterpillar. When larvae burst out of the caterpillar's body the caterpillar covers the larvae with silk and protect them until the wasps hatch.

4. The sexual part of the life cycle of *toxoplasma gondii* takes place in the small intestine of cats while other warm-blooded animals, e.g. mice, cats themselves and about 3 billion humans all over the world, serve as intermediate hosts where toxoplasma asexually reproduce in intracellular cysts in the muscles and brain. Infected rodents become drawn to the scent of cats that helps the

parasite to sexually reproduce when its host is eaten by a cat, while infection in human subjects increases assertiveness in young females and risk of traffic accidents among males. (There is no consensus on whether attraction to the cat's urine smell by infected humans is evolutionary beneficial to toxoplasma.)

5. *Rickettsia*, a distant cousin of mitochondria, is a genus of bacteria that invade cells of plants and animals. Rickettsia are carried by many insects, where it can sometimes pass from mother to daughter. Rickettsia cause a variety of diseases in humans, e.g. typhus, but it may be, apparently, beneficial to some animals. For example, sweet potato whiteflies infected with *Rickettsia sp. nr. bellii* produce twice as many offspring as uninfected whiteflies.

6. More controversial instances of gene's (genome's) avatars are *beaver dams* and human-built *leaking sewage systems.* The latter, one may argue, are avatars of the genome of *phage* VPIΦ (CTXΦ according to some authors) that infects the *Vibrio cholerae* host and encodes the toxin which (along with TCP– *toxin-coregulated pilus*) causes the virulence of the vibrio that inflicts profuse diarrhea in humans. (Since bacteriophages do not directly infect humans, the viral influence on the brains of the leaking sewage systems builders remans debatable.)

7. Similarly, a dynamic fragment of humanity may appear as an avatar of a *computer* virus that infects a bug in a computer program.

Richard Gross was heading to the Kennedy airport in a taxi.

When Arnie told him about meeting a hysterical robot in Central Park a week ago, he took it for just another of Arnie's stories but Mark's call was disquieting. A robot in a loop is not supposed to spontaneously unfreeze.

R-0204J11, one of Jenkins' robots at the airport, went into a loop last night. An RB contoller saw it on his screen when he was alerted by the telephone call from JFK. Suddenly R-0204J11 unfroze and the man could hear over the phone the emergency string apparently emitted by the robot upon unfreezing.

Mark checked R-0204J11 early this morning. The robot stayed in the loop for about 10 min and was fully operational afterwards. Just before it froze, the robot was talking to a woman who told the robot her name as well as those of the town and the street she lived; also she told the robot what she thought of her neighbours. There was no record in robot's memory of an event that could provoke a loop and, inexplicably, there was fifty three second discrepancy between robot's last auditory record and the freeze cut off point.

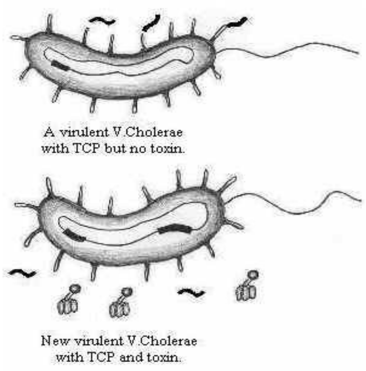
The loop was a nuisance from the users point of view, a software bug. But this bug was implanted with great care and for a reason. Richard hated to admit to himself that he did not know the *full* reason of how and why. But, in any case, only his "boys" were supposed to know how to unfreeze a robot in a loop.

Upon his arrival, Richard could not find anybody of the evening robot service crew except for the cleaning woman at "robots' quarters". She heard through the door two robots loudly swearing at one another.

– One of them shouted, the same bad words he said in the hall. Then the second, I think it was the big woman, said "shut up" – The woman told Richard.

– How do you know they were robots?– Richard inquired– Have you seen them?

– When I went into the room the two were leaving, I saw the man from



behind. He had red hair. I've been around here for two years, I can tell who is a man and who is a robot by just how they walk.

– You said the woman was big...

– Very big. That voice of hers was real loud.

– Thanks a lot –Richard said – what you've told me was very valuable – and he handed her $100 bill.

The woman smiled slyly.

– I have something twice as valuable if you...

– Let it be thrice.

The women pulled out a wrinkled sheet of paper from her pocket.

– You know, I was emptying a trash bin. You said three...

Richard smiled and passed three bills to the woman. She gave him the paper. It was a complaint form where it was written that

"one robot killed another with foul words and illegally scared Jenny with his indecent companion". Signed: Jessica Williams, the witness.

Richard thanked the woman.

– This makes it simple – He thought and dialed the number given by the witness on the form.

– Hello – he heard a high-pitched female voice.

– Good morning, madam – Richard began. – My name is Richard Gross. I am investigating the yesterday accident with the robot at the Kennedy Airport. May I speak to Miss Williams, please.

– I am Miss Williams. I am the witness. That was disgraceful! I told everything I think of this to the men who control robots this morning.

– You mean from the Robot Control Agency, do you?

– Yes, two gentlemen wearing very nice hats.

– Bloody RCA! – Richard thought – Damn it! I have to move fast.

– I represent a different agency, madam. We assess the moral damage to the passengers that need be compensated. According to your statement at the airport, madam, you are the most important witness. – Richard neglected to say where he found this statement.

– Are you willing to help us?

– Yes I am. That companion woman immorally damaged Jenny.

– By law, Madam, I am not being allowed to record statements made over the phone. If you permit, I will come to your place in twenty five minutes and you will tell me everything in person.

Crossing the arrival hall on his way back Richard met two men in Texan hats. – Ah – He chuckled to himself – RCA cowboys came to JFK. I bet, the cleaning lady has a $1 000 story ready in wait for them.

Richard hired a black shiny limousine to Williams' house where he get out of the car and knocked at the door. A woman let him in and the words poured out of her mouth.

– I'll tell you. The robot of that companion woman frightened Jenny. It is illegal. The toys must be compensated...

– Certainly, madam, all inconvenience incurred will be compensated when the full evidence has been collected. May I put on record your testimony that the airport robot was disabled by a woman illegally accompanied by an unregistered robot who damaged the toys of your daughter?

While the woman was digesting the question a boy of about six came in.

– Mommy, he also thinks I am a girl.

– You are not a girl, my dear – and, turning to Richard,

– The first robot said I was a good mother. And that indecently fat ugly woman... –

Her lips curled scornfully.

– She was not ugly, she was a companion – the boy interfered.

– You see – Miss Williams turned to Richard– what she's done to my Jenny.

At that moment a man with a guilty face entered the room.

– They could not find the green bag, darling. It had no luggage sticker on it.

– I knew it Tom, I told you, seven...

– You said six, mommy – The boy interrupted her.

– I am sure, seven. Now all you toys are lost, honey –

– I do not want these toys – whined the boy – I want a robodog

– We can not afford it honey.

– Then I want a companion.

The woman looked at Richard seeping with indignation.

– You see, the boy is immorally damaged.

– Yes, I see – Richard admitted gravely – Nevertheless, I need to ask your son, who is the damaged party, a few questions. I must inform you that no one except for the person conducting the questioning session should be seen or heard by a witness during questioning lest the validity of the testimony be contested and no compensation be issued. But you can listen from the adjacent room where you are advised to remain silent.

– What kind of compensation– the woman inquired suspiciously.

– It will be a new set of toys for your son.

– I want a robodog – said the boy stubbornly.

– Jenny, if you tell the truth, I promise it will be a robodog, the latest model.

Richard smiled to himself – *I* can afford it.– He liked the boy.

The woman hesitated.

– And – Richard turned to the woman,

– You will be personally informed by a court letter on the penalty imposed upon the guilty party when they are apprehended and brought to justice.

– OK – the woman said – Tom, let's go over there.

The two left the room.

– What were the last words of the robot who spoke to you mother before he stopped moving – Richard asked the boy.

– Robot said that Mister Jenkins had bare hands. But he was not real and the man with red face wanted to break his hands. Then the real robot said to him ...

– You do not have to say it Jenny. Tell me, what happened in the small room?

– I asked the real robot to show me how he did it. He said it again. It was very loud. But he was cheating, he opened his mouth.

– Are you sure, Jenny?

– Yes, first time he did not open his mouth and it was not that loud.

– God – Richard thought – How loud it must've been for the second time if 130 dB was not that loud.

– What happened there to the first robot?

– He shut up.

– Why?

– Because the real robot said "shut up".

– And what do you think, Jenny, was the real robot's companion also a robot?

Jenny thought for a few seconds.

– No, she wasn't a robot, but she was real.

– Thank you Jenny, the robodog will be sent to you in a few days.

– I will call him Robi! – the boy was delighted.

The parents came back to the room.

– Thank you very much Miss Williams, your testimony and that of your son were most helpful. I must leave now. May I ask your husband to give directions to my driver?

When the two men went outside Mr. Williams said reproachfully.

– You shouldn't have promised that to Jenny, he believed you.

– The robodog *will* be shipped in a couple of days.

– Are you kidding? One can buy Porsche for this money.

– Courtesy of our agency.

The man opened his mouth.

– But remember – Richard continued – Jenny must be left alone talking to the "dog" for the first five minutes. *Then* it will be *his* dog, taking commands from him alone. Get the idea?

– I do, but...

– Indeed, there may be a problem, there is a bug in our mailing system and a shipment is sometimes diverted to a customer's business address. It will help if you give me yours.

The man's face has brightened with a smile.

– Then it will be no problem – He said pulling out his business card.

Richard took it and replied opening the door of the car.

– Some problems *will* come later, but the dog is programmed to handle certain... problems.

He paused before getting into the car and asked.

– What happened at the airport? What is your impression of this real robot and his companion?

– The man was powerfully built and had intensely red hair– Mr. Williams answered– the woman was a tall blonde... very good looking – he added conspiratorially almost in a whisper.

– Was the woman fat?

– Not around the waist – the man smiled and continued.

– There was something... well, real, as Jenny says, about them. There was a heavy red faced man next to them who spoke with a Southern accent. He asked the robot, who was talking to my wife, who had made it. The robot answered that he was created by Mr. Jenkins. The man said he wanted to break the hands of the creator and then he swore. Everybody around was stunned, it was very loud, but the blonde laughed and said something to her companion. I am certain that the red haired man kept silent all along, I saw his face. Then the red faced man called for police.

– Was your son scared?

– Not at all. He was delighted – The man smiled.– He keeps nagging me that I should be saying it to mother every so often.

– Thank you Mr. Williams – Richard said and added getting into the car.

– The robodog will be fun for the father as much as for the son.

While driving back to the city Richard recalled boy's happy "robi" and smiled.

– Money may have its uses – He thought.

– It was a luck talking to so young a boy who saw what happened. Children's brains are hard to deceive by illusions. Immature synaptic connections have their advantages. Who are these "real robot" and his "companion"? Can Arnie and his new buddy Ryan, these two big never grown up kids, be of any help? Richard appreciated their childishness; he knew the price the brain pays for maturity. But no matter what he may learn from them– he realized – the mystery of the spontaneous unfreezing will remain.

–Yet, there is nothing to loose, I will call Arnie tomorrow– he decoded.

When Richard arrived at his office in Manhattan his secretary told him that Mr. Gibson was trying to reach him over the phone and said it was something urgent.

Gibson was the deputy director of the Robot Control Agency. This agency recently sprouted out of the powerful Robot Prohibition League that united such unlikely allies as Religious Right and labor unions. The federal government, trying to take it under control, created RCA which absorbed many "moderate" RPL activists that were now working under government bureaucrats. New York, Massachusetts and California did not fully recognize authority of RCA.

Richard was always wondering how Gibson, being *obviously* incompetent, could have been appointed to that position. There was an apocryphal story about the man and a robot that was programmed to classify people's characters by the tone of voices and body movements. Gibson, at an official demonstration of the program, asked the robot.

– Hey, you, boy, what do you think of *me*?

– You are an asshole, sir – The robot answered.

– What?!

– A stupid, incompetent person, sir, according to the Webster dictionary, sir.

– How dare you?!....

– Records of you public appearances were in my neural network training set, sir, you are an exemplary asshole, sir, deserving imitation, sir, according to the Webster dictionary, sir.

When Richard connected to Gibson's office the man shouted.

– Are you aware what your robots are doing, Gross!? Our agent at Kennedy reported that one robot was making blasphemous statements, the second was loudly swearing in the public place and the third, a woman robot, was attempting to seduce minor.

– Congatulations, Mr. Gibson, your agency is very efficient. In so short a time you found out that the red faced man who, according to several witnesses, shouted first the blasphemy and then the profanity and, thus, immobilize an RB robot was a robot himself and that the woman, Miss Williams is her name, I believe, was also a robot. Have you already informed the New York police? I presume you have an overwhelming evidence in this case. What was the name of that robot suspect last time ... yes, O'Connor.

– The man was red haired, not red faced and it was another woman. – Gibson objected with feigned confidence.

– Probably, the light was poor – Richard replied – Three witnesses say the suspect was fat, red faced and had a Southern accent. He said he wanted to brake the hands of the Creator and swore. He deceived the security men by pointing finger at an innocent bystander, who, indeed, had a red hair and, who, according to several witnesses, had never opened his mouth. I do not have any reason to believe the fat man was a robot, but our company, as well the New York police department, I believe, will be most grateful if you provide the information available to you about the man.

Richard could almost see the man gasping. It was their agent, probably, from the old RPL guard.

– You do not have to be a robot to read the mind of this man – Richard thought.

– And, according to the visual record of the airport robot and several witnesses, the only woman in the immediate vicinity of the minor was Ms. Williams, minor's mother. If you officially maintain that she is a robot, Jefferson Kerr, will be only too happy, I believe, to represent her. Remember, the lawyer who won the O'Conner moral damage case.

– Gross, I will not leave it at that! – Gibson shouted. – I will find this redhead and his robot companion and ...

– This will be most appreciated Mr. Gibson. These two may provide an invaluable evidence on how the airport robot was damaged. Please let me know when you find them.

– Do not overunderestimate me, Gross. We shall twice outsmart and outfake you human-fake robots, we shall hunt down every one of them.

– What do you mean Mr. Gibson? – Richard sounded very worried.

Do you really plan to employ men disguised as clandestine robots who impersonate humans? Instead of making a suspect feel safe, this may send an RB robot into an irreversible loop, frozen for ever, especially if one says something threatening or horribly blasphemous about the creators of robots' personalities, Jenkins and Hamilton. It would be unwise to pursue your idea to its fruition, Mr. Gibson. Besides, checking the serial number of a frozen or unfrozen suspect under the shirt does not necessarily tell you if it is a *clandestine* robot.

Richard could almost hear Gibson's mental gears turning. After a pause the man shouted triumphantly.

– Do not tell *me*, Gross, what to do with *my* ideas! *I* decide myself what I shall do.

After the conversation, Richard had a second thought whether it had been wise to make Gibson mad at him. The man excised a significant power, even within the New York state.

– Well, ... being angry, he will behave even more stupidly than usual; yet, it would be good to find the "real robot" and his "companion" before Gibson's people did and to warn them – Richard thought.

– But the two can, probably, fend for themselves, they were "real" – He chuckled.

.

6.4 Taxonomy Problem.

Are you and your fiend X in the same clade of the phylogenetic tree? Maybe X is a queer quirk of the canine convergent evolution to human-form?

What matters: genotype or body shape? Are crocodiles featherless birds rather than cousins of iguanas as cladistics tells you?

In 1942, Ernst Mayr, a leading 20th century's evolutionary biologist, following a suggestion by his senior colleague Georges Buffon (1707-1788), came up with the following idea: Interbreed with X and check for grandchildren. If there are some, you and X are of the same species.

But what if X, even if of the opposite sex, is busy with a similar experiment of his/her own?

Simple – this is the 21st century: scratch, better surreptitiously, a sample of your friend's skin, sequence DNA and count the matches. 1,2,3... done.

Damn it – 98.5% dog – this X is *just* a talking animal! *Now*, since I know it, he/she'd better not forget to wag his/her tail at me, OR ...

This "OR" of yours is shit and 98.5% is *just* silly statistics. It isn't a %, say, in chimpanzee's genome that allows their bodies to navigate trees with 95% agility and to hunt monkeys with 75% efficiency. Besides, dogs make best friends with 97% probabilty.

But X is not quite a dog. You impinged on his/her privacy – he/she becomes mad at you. Your best friend is gone, you will not see him/her again.

Nevermore – 100%.

Eva was hurrying to her lab with Jason's copper hair in her pocket. She knew what to look for. With her multi-scale nano-analyzer she could have the full structure composition of the hair by the lunch time.

The elevator was stuck as ever on the 5th floor. Impatiently, Eva was running up the stairs to her seventh and saw, horrified, four men in blue overalls shoving, not too gently, her analyzer down the staircase.

Panic-stricken she could hardly talk.

– Careful! My analyzer! Who are you!?

– Professional movers – responded a big baldish man sweating and panting.

Eva could hardly make out an unfamiliar soft accent.

– Not break thing – he added reassuringly giving the analyzer a healthy kick.

– Your? – He pointed at the analyzer.

– Yes, but why!?

– Instruction, all seven move – The man pointed upstairs.

– But why? and why do you carry it downstairs then?

The man explained

– Very bad corner on seven, thing not go – he made a turning upside-down gesture, gave the analyzer another kick and added reproachfully

– Very heavy, very bad hold, very very bad corner.

– But it came that way, just four month ago, what happened?

– Not know, instruction.

Out of breath Eva stormed into Mike's room.

– What the hell is going on!? Why...

Mike, their administrator, was ready for this.

– Calm down, calm down Eva, everything is under control –

he said in a tone one pacifies a patient who's just learned the difference between melan-choly and melan-oma.

– Doctor Kerr...

Eva interrupted him – Nobody told me...

– Why?... I sent you a memo two months ago. Look –

he turned the screen toward her

– you thank me for informing about the renovation in advance.

Eva froze. The "thank you" mail program was everybody's anti-memo shield. Did Mike know about it? Time to change the subject she decided.

– Sorry Mike, I forgot it. But why do they go via the sixth floor?

–Oh, the sixth has not been renovated yet.

– So what?

– Let me show you.

Mike, beaming, ushered her along the corridor to the corner, all made in black plastic with ten inches long indescribable somethings ominously protruding from the walls.

– Isn't he a genius? The designer.

–Bloody Stupid Johnson – Eva muttered,

– His name is Jackson, Ariel Siegfried Jackson.

– Sorry, I got mixed up, thank you Mike. ... Jerry Kerr...?

– Yes, I persuaded doctor Kerr to help us. You better hurry, he agreed on only five samples from us every second day.

On her rush to the biochemistry building Eva was weighting the possibilities.

– Hmm... Jerry's laser assisted multi-mass-spectrometer, probably, will do but Jerry will not allow her any close to it. Himself, a molecular geneticist, he would find DNA at the center of a black hole – she mused – but I will sort it out – she decided.

Jerry was not in his lab, the secretary next door greeted her.

– Morning, doctor Helguson, your luck is here, the last for to-day. Give me you sample. Is there a message? I will attach it along with the tag.

– Thanks Jenny, ask Jerry to call me when he is through with it.

Stepping out of the room Eva came face to face with Arnie Swift.

– Hi, Eva, the news is all over the town. Has you lab survived the hit by ... what's the name of this genius... yes, Johnson?

– Jackson.

– Bloody Stupid Jackson?

–No, that was Johnson.

– This is what I said. And what is there for Jerry you brought? Plastic? Ceramics? Microfiber?

– Some hair.

– Hair!? Cat's hair, dog's hair, elephant's hair?

Eva, to her own surprise, felt reluctant to say the truth.

– I would rule out elephants. Sorry, Arnie, I must run and supervise the installation.

– Good luck and do not forget about the dinner.

Jenny smiled at Arnie when she saw him.

– Too late, Arnie, doctor's Helguson was the fifth and the last one for to-day.

– Is it this little plastic bag?

– Yes this is hers.

Arnie's eyes sparkled with hidden laughter.

– Look, Jenny – he exclaimed,

– See, the eagle, at the wall near the window to your left.

Jenny leaned out of the window.

– I do not see any big bird there.

– It is a baby eagle, there must be a nest somewhere on the top of the tower over there.

– Looks to me more like a pigeon. – Jenny glared at Arnie with suspicion.

– Come on, Jenny, I do not need, as your boss does, a mass spectrometer to tell a pigeon from an eagle–

And smiling mischievously he inquired

– How is the new automatic cleaning system in the lab?

– Doctor Kerr complains that samples started to spontaneously disappear the first day it was installed.

– How unfortunate, but... samples often disappear.

An hour later Jerry showed up.

– Hi, Jenny, How much's there for to-day?

– Five samples.

– What a bore. Please, Jenny, give them me one by one.

– Too early for lunch – Jerry murmured emerging from the lab yawning an hour later. – Is there something left, Jenny?

– There, the hair, doctor Helguson brought it this morning.

– Hair? Was somebody murdered at the Chemistry?

– I think its an animal, I overheard doctor Helguson saying to Arnie she was not certain. Probably, a dog or a cat.

– Arnie was here? He could tell her by just looking at it. Why, on Earth, does she need my mass spectrometer to decide between a live dog and a dead cat? Quantum entanglement, I presume. Anyway, it will only take a minute.

And Jerry closed the door of the lab behind him.

Two hours later he emerged again all trembling with excitement.
– What happened doctor Kerr?
– The hair! The damn system sucked the remaining samples!
– But you, probably, have figured out what animal was that, You said only...
– Jenny, no animal with this hair can conceivably exist.
– An artificial hair?
– Not with to-day's technology. Do you have Eva's cell number.
– You hold it, doctor Kerr, she wrote the number in her notice.
– I forgot about it. Oh dear, *she* is curious about the 3D arrangement of this hair. *I* want to see the animal.
– You said it can not ...
– My glasses! Where are my glasses? The damn systems must've sucked them along with the hair. Jenny, please, dial for me.
– You just put the glasses into you pocket.
– Thanks god! But please dial it for me anyway.

Eva was fighting with a tomato sandwich when her cellular rang. She extracted the phone from her right pants pocket with her moderately clean left hand. Jerry's voice shouted.
– Crazy, Eva, absolutely crazy, bloody sucker!
– Bloody Crazy Sucker? Slower down, Jerry. How is my sample?
– Gone. The cleaner sucked all it in.
– Before you...
– Not quite, I started with polynucleotides, naturally.
– Naturally.
–Lots of mtDNA, and as you had guessed...
– Me? Guessed?
– Don't interrupt. How did you come across it?
– At the plane when...
– Which company? We can trace the owner...
– You mean... the owner of the hair?
– Put it this way if you wish.
– I have the e-mail address.
– Which country?
– He lives in New York.
– Fantastic! Can you arrange another sample and let me see the body that comes with this hair. Was it alive, by the way?
– Dead bodies don't fly, Jerry.
– Furry dead animal skins do, first class, but never mind. Can we make it this afternoon?
– Why such a rush?
But she was smiling to herself at the idea of seeing Jason.
– If he is on-line,... Where do you want us to meet?
– The South pole is OK with me... The doggy place at the Square may be even better, if he lives close by.

The answer came in ten minutes, Jason could be there within an hour. Eva smiled internally again. She called Jerry.
– 3:15, where you said. Explain to me...

– Not now, I am after new matches – it's unbelievable! See you– he hang up.

The three of them converged at the Washington Square almost simultaneously.

– Hi, Jason, this is Jerry Kerr, the biochemist I wrote to you about. Jerry, here is...

Jerry, impatiently, interrupted her.

– I am not in a mood for seek and hide, show me the animal. Which one...

He pointed toward the canine playground with a dozen of dogs chasing there one another.

– What animal? – Eva gasped in surprise

– The owner of the hair, as you called it – he said giving Jason an angry stare.

Jason started turning red.

– Look, Jerry, I never said I thought...

– Of course, you thought that was a secret love child of a golden retriever and a red tomcat, very romantic. Enough of this buffoonery. Where is the animal?

– You got it wrong Jerry, that was human hair.

Nearly paralyzed with anger Jerry stuttered,

– You... you tell me, *me* I confused the primate and canidae mitochondrial sequences !?

– I don't know – Eva responded, also growing angry – but this was his hair – she pointed to Jason.

– Don't tell me he is a cynocephalus –

Jerry responded irritatedly and, turning to the young man,

– Assure this woman your mother was human.

A visible wave of fury rolled over Jason but he gained control. He muttered something, hardly moving his lips, almost in a whisper, calmly and slowly. In a second, he was gone.

Jerry turned as white as a ghost.

– You did no tell me he was Russian.

– Not by his name.

– His name? You've not...

You did not let me to, it's Jason Flynn. He was terrifi...yng. Scared the hell out of you.

– Scared? I was about to hit him.

– You? Hit *him*? After what happened? You did not even understand...

– Yes, I did. My granddad on mother's side – a virtuoso of Russian profanities, taught me to swear in Russian. When I was about four, I used that on the grandma...

– You called her a mitochondrion, didn't you?

Your friend improvised.... But why insulting me so obscenely? Why in Russian?

– Damn, Jerry, *you* started it. What you said about his mother...

– I did not say anything disrespectful, you did.

– Me? I hardly opened my mouth.

– You insisted it was *his* hair with the canidae mtDNA in it, implying he had a canine for a mother.

– I never...

–You said to Arnie it was either dog's or cat's hair, Jenny told me.

– All I said it was not from an elephant. *Arnie* brought up this dog business.

– Bragging about his Gregory, I presume.

– I ran away before he got started.

– Smart girl. He keeps nagging me to make a full genome map for his dog as if I have nothing else to do. And why do *you* come to me with this hair?

– It is unnaturally iridescent – must be a peculiar keratin matrix. I am not making it up, Jerry.

– I did notice iridescence in your sample... Was there a dog on the plane next to you?

– Not even in the pilot seat. No canine DNA could creep in. Better say what BLAST you currently use.

– 3000. Very fast BLAST.

–That's what I thought, it is buggy.

– Not that buggy, I presume, to find bug's strings in dog's sequences... Why, on earth, in Russian?

– What was that he said in Russian?

– You better not know.

– It can not be worse than Richard's emergency string. I am immune.

– Your string is a baby toy, measles shot against bubonic plague... Why did he swear at me in *Russian*?

– He's got a knack for accents.

– What accent? I do not speak the language, not a word for sixty years.

– You said when...

– Do you think the man could lip read the imprints of my childhood memories? This is like Arnie who believes his Gregory can distinguish colors and read Sanskrit. Who is this man?

– I just met him yesterday on the flight to New York. Tell me, your system keeps the oligos data, does it?

– Of course it does.

– Can you check the matches with another program?

– What's the point, your sample was contaminated. If you go for another one...

–Try it yourself, Jerry: jflynn at gmail.com. Good luck. Ciao.

6.5 Babel Chamber.

Mara, the demon of Mundane, who had tormented Buddha and was confined by his daughters to a magical chamber under an Agasthyamalai Hill at the southern edge of Sahyadri Mountains, was believed to be able to sustain a twenty four hour discourse on any mundane subject in all languages with anyone who had a misfortune of having entered the chamber.

For more than thousand years philosophers had been debating if Mara *truly* understood all languages or only the Old Tamil. A single language would suffice, some argued, since all strings of words in all languages had been already written in the Book of Mundane and instructions written in Old Tamil for finding a correct page, would suffice for the demon. (Mara would not dare use the sacred language of Sanskrit – everybody agreed on this.)

The debates stopped when the great 12th century mathematician and astronomer Bhaskara II had demonstrated by an elaborate calculation that the Universe was not big enough to contain such a book. Generations later his calculation was lost, even called "mythical" by some philosophers and the ancient controversy started anew. It became most dynamic at the turn of the 21st century running ripples on the water of the two mainstreams of the Western thought – pluriholism and psychomonism.

Amazingly, the abstracted analysis of the concept of "*true* understanding" by modern philosophers happened to be relevant for understanding a mundane life event.

When Gregory brought me to Arnie a couple of months ago, we instantaneously, which is rare for people after certain age, became close friends, free and relaxed in each other's company.

This is how it happened. I was crossing an island of green lost between high rise apartment buildings in the Village and, suddenly, saw Gregory walking toward me. I froze fascinated by the impossible perfection of the lean muscular body and his smooth predatory gait. He stopped at a distance and sat down. I took a step closer to him and lowered myself down on the ground.

– How come you are walking alone, where is your partner?

I said "partner", not "owner" deliberately. This magnificent creature was not to be owned by anybody – I saw it right away.

He stood up, turned around, went back a few steps and whistled. I opened my mouth in surprise hardly paying attention to a man who appeared from behind a thick Azalea bush.

–Hi – The man smiled at me – Gregory thinks you are OK. He is never mistaken. My name is Arnie. What is yours?

– Ryan – I responded rising to my feet. How can he tell who is OK, by whistling?

– Dhole blood. He reads you by how you move. Most of what he sees is inaccessible to the human eye. And he despises people who are scared of him.

The man was immensely proud of his dog. I ventured to be nice.

– *I* would be scared if he decided to turn against me.

– No, you wouldn't – Arnie said matter-of-factly – You would be dead before you had a chance to get scared.

I laughed –

– Come on! He does look very strong for his size, but I am three times heavier.

Arnie looked up and down at me appraisingly.

– Four and no fat. Gregory, will you give a quick kiss to your new friend?

Gregory was resting on the grass a few feet to the left of me. Next second he was in the air, his fur brushing against my face. He landed a dozen feet away and turned around "smiling" at me – his tongue hanging out. Bewildered, I brought my hand to my nose moist from his "kiss".

Arnie beamed at me triumphantly.

– If he went for your jugular he could do it twice as fast.

I could not but agree, I was ready to believe everything about Gregory. But as other Arnie's friends, I figured out how Gregory "reads" Tamil; by a tacit agreement, this was never mentioned in Arnie's presence.

I was heading to Arnie's apartment for a dinner party wondering why, besides Victor, Frank and myself, he had invited such unlikely people.

Helen Hamilton and Jeremy Jenkins were "designers of sissies" at Robot Beautiful. Arnie did not hold them in a particularly high esteem, but, I chuckled inside, Gregory had decided otherwise.

Eva Helguson, by what I understood from Arnie, could go along with Frank and Victor but she did not meet Gregory before. An invitation for diner without Gregory's OK was something new. I chuckled to myself again.

Victor Boribeda was a long acquaintance of Arnie, a freelance scientific journalist with whom Arnie had travelled together to remote places. It was Victor who had arranged a fake certificate of health for bringing Gregory from India to US. Gregory and Victor loved each other.

And if there was little warmth lost between Frank, a soft matter experimentalist, and Gregory, their views on humanity converged. A human for them was a biped with the sense of smell of a congested pug and the vision in physics of a retarded bat.

When Arnie and Gregory welcomed me at the door, I heard Frank's voice:

–...and who do you want make to believe that the women who tried to seduce Buddha were robots!?

– Many male readers of my column got it wrong – Victor answered.

– They are bombarding us with letters asking where one can purchase such robots. All I said was that the synergy of Mara with the daughters of his spirit, regardless of their physical non-existence, could be reconstructed similarly to the algorithms controlling RB's robots.

– Even funnier – Victor continued– some complain that Mara had plagiarized their ideas. For example, a graduate student from Berkeley says that his philosophy professor had clearly stated in a famous article thirty years ago that not the physical existence but the existence *in principle* is essential for synergetic emergence of syntactic understanding. He threatened to sue the New York Times.

– Existence of what? Of more lavish details of how these daughters seduce Buddha in your article!? And – turning to Gregory – why does your friend keep writing such rubbish in the *Science* Section of this tabloid!?

– It is Scientific *Philosophy* and my Understand "to understand" column has the highest readers' feedback.

– As high as the toilet paper controversy?!

– I meant science.

– But you said "philosophy", doesn't this go with the toilet paper?!

– What's wrong with this? You recently published an article in a toilet paper tabloid yourself.

– Me?!

– Wasn't it *you* who called Science and Nature toilet paper tabloids?

– But...

They stopped arguing when I entered, this performance was not for my ears but for the sake of the novices, Helen and Jeremy.

The reaction of these two, however, was disappointing – no sign of a shock, not even a surprise. What else could you expect – these are *Arnie's* friends was written on their faces. Gregory had a better understanding of humans than his zoo-psychologist partner.

The pause allowed Helen to partake in the conversation.

– You are too harsh Frank – She said quietly in a deep low voice.

Frank's face lightened.

– But these philosophers, being themselves accomplished ...

– This is OK with me, I do not know much philosophy anyway. I mean you have a blind spot for the toilet paper.

Frank opened his mouth, he did not expect this kind of a gambit. Helen continued.

– When, Frank, you understand something hard in physics it triggers in you a particular feeling of exhilaration. Since you cherish this feeling, it makes you passionately angry at those who say "I understand" without ever have experienced such a feeling themselves. But some get similarly exhilarated when it strikes them how to correctly put the toilet paper on the wall. Is it important whether you get drunk with champagne or with beer? And, turning to the dog

– Do you agree, Gregory?

All this was said sweetly and sincerely. If Helen was teasing Frank, neither the tone of her voice nor her posture betrayed a shade of irony. Before Frank could decide how to react Jeremy interfered.

– I want to say a word in defense of the modern philosophy. Its position with respect to science is similar to that of religion with respect to the concept of God.

Frank exploded.

– Science has nothing to do with your God that does not exist anyway!

Jeremy, obviously, expected this.

– Have I made any statement about the existence of God, about science, or even about the *concept* of God? I was about to say that as much as religion tells people that there is something beyond their mundane lives with the reference to the concept of God, the scientific philosophers, and scientific journalists for that matter, do the same with the reference to the concept of the scientific truth. Who, do you think, expect pastors to believe what they preach and philosophers to understand what they say? Do my robots understand when they say I am their creator?

Jeremy, like Helen, spoke quietly and seriously, with no trace of cynicism or mockery to his voice.

Frank, who was desperately struggling for an appropriately outrageous answer, was saved by a ring at the door.

Eva came in. She shacked hands with everyone and tried to pat Gregory's head. Her mind was elsewhere. Gregory moved his head away and briefly sniffed her right hand. He looked puzzled. He sniffed the hand again, longer this time, then went around sniffing her feet and legs and eventually concentrated on her left hand. He looked even more puzzled.

– He is trying to recall something – Arnie said.

– Didn't you recently shake hands with somebody from around Hyderabad?

– Hyderabad?

– We came across Gregory with Victor in the outskirts of the city. Your hands may keep the smell of somebody he knew there.

– I have not seen today any visitor from India.

– And some days earlier?

– How much earlier?

– Depends on how often you wash you hands. Say, within a week.

– Why such a low opinion of my washing habits?

I interfered

– Arnie, do not exaggerate. Gregory's olfaction is incredibly good. But even a perfunctory hand washing reduces the number of molecules of whatever, say, tenfold. Do it twenty times and you hardly have a single molecule left. Right?–

I turned to Eva, – You are a chemist.

Arnie laughed.

– These mathematicians. Have you ever tried to wash off skunk smell?

– Arnie is right – Eva said

– It may stick to the skin. But why do you speak of Gregory as if he understands something. He is just a dog.

It suddenly grew quiet in the room. The sweet low voice of Helen broke the silence.

– Do not pay attention to what she says, Gregory. She is just a woman.

Uncertain whether to become angry or to laugh, Eva gave a sigh and managed a smile.

– Sorry, I forgot, I know, he reads Sanskrit.

– Sanskrit? – exclaimed Arnie in a theatrical surprise. *I* did not know this.

– Eva said defensively.

– Jerry mentioned it today.

– Do not trust him on that! The other day he told me that he had found the original Sanskrit version of the Little Mice and the Big Elephants Panchatantra story in the genome sequence of Indian elephant. But then...

Victor interrupted him. – Enough is enough, Arnie – and, smiling at Eva –

– Gregory can not read Sanskrit, at least not in the Devanagari script, but he recognizes the *Tamil* words on the cards that are attached to his toys, a few dozen of them.

– Tamil...? I do not...

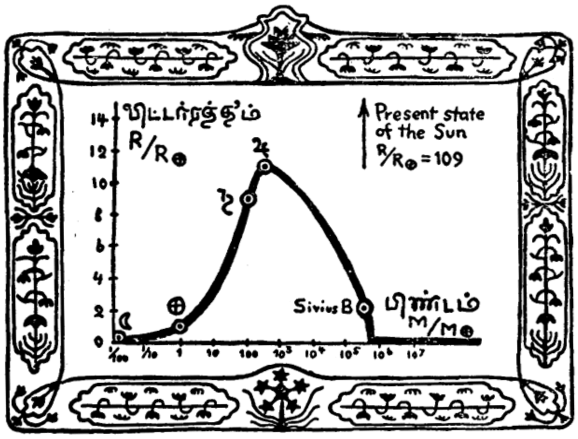
– Raman, Chandrasekhar, Ramachandran – Frank interrupted her.

– I did not know they were Tamil. But do not tell me that Gregory reads it with a Raman spectrometer. How...

– Do you want me to show how he does it? –

Arnie said and a transparent box with a dozen toy figures of animals magically appeared in his right hand.

– These are for kids learning to read Tamil. There are cards with the written names of the animals magnetically attached to the bottoms of these toys. You



Ill. 62. The relationship between the radii and the masses of cold stellar bodies, according to the calculations of the Indian astrophysicist S. Chandrasekhar. The symbols ☾, ⊕, ♄, and ♃ respectively represent the Moon, the Earth, Saturn, and Jupiter. Note that for masses greater than 460,000 times the mass of the Earth, the radius becomes zero! The words for mass and radius are in Dr. Chandrasekhar's original Tamil.

show a card to Gregory and he will pick up the animal. And for the sake of those of you who can not read Tamil...–

An identical box appeared in his left hand.

– Eva – he said with the two boxes in his hands – I forgot to ask you. Has Jerry found something interesting in you hair sample?

– The one who found it *very* interesting was the automatic cleaner in his lab. The damn system sucked in the hair.

– Arnie opened his mouth in disbelief and dropped the boxes. The little figures went all over the floor. Hastily, he kneeled down and put them back to the boxes.

– What a shame – he said rising from the floor.– Do you have more of this hair?

– No. And no way to get another sample, thanks to Mister Kerr.

– Jefferson Kerr? This notorious New York lawyer? – Jeremy inquired.

– No, I speak of a geneticist, Jerry Kerr, actually, he is Jefferson's brother. Different vocation but the same killer instinct.

Arnie looked amused.

– Do you want to say, Eva, that Jerry killed the animal and then pulverized all its hair with his mass spectrometer? Didn't you say it was an elephant? How has Jerry managed doing this in one day?

– It was human hair – Eva answered somewhat reluctantly.

– Then, I see no problem.

– Jerry insulted the man. He said something to the effect that he, Dr. Jerome Kerr, despite all the evidence to the contrary, strongly believed that the mother of the man had been human rather than canine. I do not dare to ask for another sample.

– Canine?! What evidence?

– Something went wrong before the hair was sucked in. Jerry did not realize that and sequenced what he thought was mtDNA of the hair. He got very

ANIMALS

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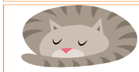
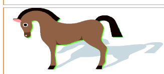
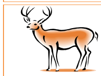
 பூனை	 நாய்
 யானை	 குதிரை
 மான்	 சிங்கம்

excited and wanted to look at the "body", probably, expecting to see a dog with butterfly wings. When he realized he had been mistaken, he turned... well, his usual self.

Helen interfered.

– The man with this hair, is he your friend?

Eva hesitated.

– I met him yesterday on my flight to New York.

– And he offered you a sample of his hair for sequencing! Why?

Eva blushed.

– It was not meant for sequencing. The hair looked unusual, I wanted to see the keratin matrix.

– Consider yourself lucky that the sample has disappeared – Jeremy said.

– The man might sue you for examining his DNA without his written consent.

– He is not *that* kind of a man.

– What do you know of him besides his hair? – Helen asked

Eva answered reminiscently bending her fingers one by one.

– Naive as a child, gifted for languages, explosive like a volcano... . Scary like hell when angry.–

Her eyes slid from Arnie to me with a hidden half-smile.

– It is for the best the hair is lost, he will be less angry at you when he learns it is not in somebody's else hands – Helen said.

– At me? It was Jerry who... . Oh!... I'll find him and apologize – Eva said with conviction.

– Now – Helen addressed Arnie, – Let us check on Gregory's Tamil. What is in the second box, Arnie?

– It is another set. When Gregory brings you a figure you find an identical animal in there and see if it is the same word on the card.

Arnie emptied the first box on the table.

– These are domestic and wild animals of Tamil Nadu in South India. Unlike Gregory, you, probably, do not recognize some of these.

– Is this a kind of a mouse?– Helen asked.

– This is Anathana ellioti – Madras Treeshrew. Very smart – 30% brainier than small rodents and adult humans per body weight.

Helen disengaged the cards from the figures.

– Where do we put the animals?

– Somewhere on the floor. OK, now show a card to Gregory, he knows the game. I will go to another room.

Gregory has hardly looked at the card, and brought a goat figure in a second.

– This animal is recognizable – Helen said, took the goat figure from another box and looked at the card underneath.

– The same two funny letters. Impressive!

Helen repeated this with two other cards. Gregory was bringing the correct figures faster than she could manage checking the words in the unfamiliar script.

Eva, meanwhile, was carefully examining a card looking at both sides of it and at the edges. Then she took another card and also one from the other set. She looked at the two briskly, smiled and sniffed the first two cards.

I knew she had guessed. Of course, she could not feel the smell, her olfaction was no match for that of a dog, but she was a chemist after all.

Then something strange happened. Gregory was sniffing and sniffing the fourth card that Helen was showing to him. Then he carefully sniffed all figures

on the floor, returned to the table took the card from Helen hand in his mouth and went to the room where Arnie was.

– You can do it Gregory – we heard Arnie's encouraging voice, just read carefully what is written here.

Gregory returned in several seconds, picked up the figure of the wild boar from the floor and Helen checked for its correctness.

– Isn't it obvious? – Eva said – the figures and the corresponding cards are marked by identical scents. Something went wrong with the boar card and you had to refresh it, Arnie.

Arnie made a half-hearted attempt to object but we all knew Eva was right. Helen smiled. Victor rose to the defense of Gregory.

– You hardly distinguish the words, you are unfamiliar with the objects they symbolize; yet, you are condescending toward Gregory. The objective meaning of word-symbols can be uncovered by a spiritual sense of smell irrespectively of a language and Gregory's intuitive power of olfaction dwarfs your syntactic proficiency as much as Mara's...

Frank interrupted him.

– We've already heard this:

"synergetic emergence of Mara dwarfs the sum of the parts of his daughters". But who dwarfs who? A profound statement written backward remains equally profound.

– Why so sarcastic? – Victor replied – Negation of rubbish *is* rubbish. But think of philosophy as music. If you play a piece in reverse...

Helen and Jeremy seemingly enjoyed this verbal fencing at the table but Arnie and Eva were deep in their own thoughts. I was sitting on the sofa absentmindedly playing tug of war with Gregory. From time to time he would stop playing and give a long look on somebody at the table. I felt he understood something that I was missing.

I was half asleep in my bed when the telephone rang.

– Unbelievable!– Arnie's voice exploded in my ear.

– Lots of what you say is, Arnie.

– I am not joking, Ryan. I am still checking. Can you come tomorrow morning?

– Morn...

– As early as you can.

–What hap...

– Gregory does read Tamil!

Next morning, Arnie excitedly started right at the door.

– I can not believe it!

– Neither can I, Arnie.

– But it is true, he does it without smell. Only one set of pieces was scented but he recognizes unscented words as well.

– But how *you*, not being quite a dog, can tell which are scented and which are not?

– I marked the scented pieces with little dots but I could not find one on the boar card after you've left.

– Come-on, you just took the wrong card.

– Unlikely, it was the one Gregory brought to me, the only one in my pocket. And the card in the unscented box *was* marked. Obviously, I put the marked boar with the unscented set after I dropped the boxes.

– Probably, some "boar" scent sticked to your hands. This is how Gregory was able to find the piece.

– Of course, I thought of that. I cleaned my hands and the card with alcohol and repeated. Gregory picked the boar just the same.

– But you told me yourself, Arnie, some smells are very sticky.

– I may be not quite a dog but I am not an ass either. I checked with other pieces. Gregory was finding the correct figures for all of them, although it took him longer with the unscented ones. But what convinced me was that from some moment on he was hardly sniffing, only *looking* at the cards.

– Well, I guess you were occasionally matching unmarked cards with marked pieces and vice versa. Or, do you remember Clever Hans? Gregory was guided by your involuntary body movements. He'll beat any horse on that.

– I know, I know, I thought I was turning crazy. I had to force myself to stop and to go to bed. Let us do it together now.

We printed afresh Tamil names of several toys from Gregory's room, went to another room and showed the words to Gregory one by one. We did not see the toys and could not give any cue to Gregory; yet, he was faultlessly bringing the correct pieces, obviously enjoying the game.

I felt even more exhilarated than Arnie was. Arnie meanwhile took a telephone receiver and started dialing. Without thinking I snatched the receiver from his hand.

– What are you doing!– he cried out – I just want to tell Victor about it.

– Don't!

– Why?!

I couldn't say myself "why" but I felt certainty inside.

– Do not tell this to anybody!

He looked at me in surprise. Then it dawned at him.

– It may be worse than you think, Ryan – He said ruefully.

I put down the receiver.

– What do you mean.

– It was Gregory's hair Jerry was sequencing, I switched the bags. I was asking him several times to make a genome map for Gregory. I am happy he never did. When he cools down and looks at the full data, he'll figure out it could not have been as simple as contamination of the sample. Knowing him, he will not rest until he finds out what happened. Sooner or later he will suspect

I was involved, his secretary saw me. It was easy to fool her, not Jerry. He will ask for Gregory's DNA and if I refuse, this will make him even more suspicious. Then nothing will stop him.

The telephone rang. Hesitantly, Arnie brought it to his ear.

– Oh, it's you Rick – He sighed with relief.

– Talk to us? Same place? I will check with him. OK, see you at seven. Bye.

Arnie turned to me.

– This was Richard, he wants to ask us something about the robot in the park. At seven, at the Trattoria. OK?

– I am free, but you said Richard had not been impressed by your "robot".

– He does not take me seriously, very much as you, Ryan. – Arnie paused – We must talk to him about Gregory.

– Richard?! He is a neuroscientist. These are the very people we must keep away from.

– Not him, He is an outcast in the community. He is, put it mildly, sharply critical of how and why they treat animals. He appreciates Gregory and Gregory likes him. He will do nothing that may be harmful to Gregory. I know him, we were close when we your younger. And he is rich, he knows powerful people. We do need his help.

– How can you trust him? He belongs with people from another universe, since he got rich.

– He is not with them. He is.... with himself. I have a reason to trust him. Tonight, you will learn why.

6.6 Who Made Who?

"...suppose I had found a watch upon the ground, and it should be inquired how the watch happened to be in that place; ... There must have existed, at some time, and at some place or other, an artificer or artificers, who formed [the watch] for the purpose which we find it actually to answer; who comprehended its construction, and designed its use." – William Paley, Natural Theology (1802)

Inquisitive minds of the 20th century detected a subtle flaw in Paley's argument: complex things, like men, descend from simpler ones, not the other way around. Thus, it was argued, the present day watches, are just the fittest leaves on the watch evolution tree. The unrelenting winds of time selectively blow away the watches that fail the time precision test.

But if *you* find a clockwork inside a smooth pebble or a pretty shell on a see-shore where no tree – only lichen can grow on the bare inland cliffs, would you search for a watchmaker shop at the bottom of the ocean or you look for a third alternative?

When we came with Arnie to the Trattoria Richard, already there, turned to me without preamble.

– Arnie told me you two had met a robot-like man in the park. What is your side of the story?

– Why *now* Rick? – Arnie interfered – You did not take much interest in it last week.

– Let us keep it in order. Ryan, tell me what your saw.

– Well, I have little to add to what you know from Arnie. The man was red haired and Arnie insisted only RB robots could have such particular color. Arnie asked the man something, he could not recall exactly what, something about Helen and Jenkins bare hands. The man became incensed. He roared, you know, your string, followed by a much longer string of exquisite profanities. We ran away from him.

– Two strong men, why were your scared?

– I was not scared – Arnie objected – I was ... confused.

– Come-on Arnie. Our legs perfectly knew what to do, not at all confused. It was by instinct. We ran. What is *your* story, Richard?

– I believe the same man was involved in an accident with an RB robot at Kennedy two days ago. RCA people are after him.

– May be the same *robot* – Arnie went on pressing his point – His hair did not look human. And his voce... . It was even louder than an ordinary robot string.

– Yes, louder, but why would anybody make a short fuse robot?

– A man in a wig? – I suggested.

– Hm... Richard mused – some RCL fanatic impersonating an RB robot... No, the man was real, I have a good witness to that.

– What witness? – Arnie asked.

– A six year old boy. Children, like dogs, smell out phonies. Unfortunately, boy's attention was fully on your read haired friend, he could not tell me what happened to the RB robot.

– What do you want to know – Arnie asked him.

– There was an irregularity in the loop and I want to know what had caused it.

– Wasn't there somebody attending your robot? – I asked.

– I spoke by phone to the man who had reported the loop, but the 150Db string had ruptured his memory. This seems dead end.

– Maybe Eva? – I volunteered.

– Eva? Who is she – Richard asked.

– Eva Helguson, a chemist, consulting RB since last December. She came back to New York via JFK that evening. You've met her once, Rick. – Arnie answered.

– Have I?

– A tall blond.

– Ah... tall blond ...

– What a keen observer! Usually, tall blonds go unnoticed by men.

– Stop it, Arnie, there was a tall blond accompanying our redhead. Do you think that could have been Eva?

– I don't believe so. She was at my place last night and she would spill out the story if she had one to tell.

– No – I objected – She was not in a talkative mood last night.

– Of course – Arnie said with a touch of venom – she was missing her Indian flight companion.

– Come-on, Arnie – I said trying to sound light and sincere – You can not forgive her for having seen through your trick with Gregory.

– Aren't you tired of these tricks, Arnie? It is time to stop playing the clown.

Richard said and then added looking at his watch.

– Somebody with the sense of humor of Gibson may take you seriously.

I felt frozen, Arnie also turned white.

– Damn it, 7:35! – Richard exclaimed without lifting his eyes from the watch. – But we still can make it despite the traffic, it will be a shame to miss the show. I will take care of the bill.

I opened my mouth. Arnie dragged me to the door with Richard following two steps behind. We took a taxi to the Times Square. Richard and Arnie kept chatting about Broadway shows until we arrived. We got out of taxi and pushed our way through the crowd toward the Foxwoods Theatre.

Before I had a chance to ask what the hell was happening Richard turned to me.

– Do you have your portable with you, Ryan? I keep forgetting recharging the battery in mine. I handed him by telephone. He removed the battery, Arnie did the same with his phone.

Assume I am paranoid – Richard said – But since there was something you did *not* want telling *me there* you'd better do it *here*.

Arnie looked at me. I nodded acquiescence. It was hard to deceive that man. Illogically, I felt exhilarated rather than annoyed with adrenaline surging through my body.

– You know, Ryan, why you can trust me, don't you?– Richard asked me.

– Well...

– You do not have to be polite, Do you?

– I don't.

– Why, Rick? I always keep my word – Arnie cut in peevishly.

– Sorry, Arnie, I appreciate it.

– Good – he turned to me again – good, you do not know. Now, what threatens Gregory?

– You see, Rick – Arnie began – We've just found out that Gregory is not quite a dog.

– Half-dhole?

– Yes, but the other half...

Arnie told Richard about Tamil, the hair substitution and the impending problem with Jerry Kerr. Richard listened attentively.

–You are not surprised – Arnie said.

Not much... with hindsight, regretfully... . Ryan, of course, was next to you when I called. It was prudent not confiding in me without his OK. Victor does not know, I presume.

– Gosh, Rick, how do you...

– Gregory is not the only one who can do it. Anyway, it *is* serious. We'll hide Gregory and neutralize Jerry. Meanwhile, I will run a few tests myself.

He looked at our worried faces.

– Nothing invasive, in your presence and with Gregory's consent. But no test will tell us *who* made Gregory.

– Why "who"? – Arnie asked.

Do you think Gregory is an accidental product of dog-dhole interbreeding? By what you say, and I am certain my tests will confirm it, there was a genome manipulation involved well beyond everybody's capability today.

Now, about Eva. Why do you think that was an Indian, the man she met at JFK?

– Gregory was sniffing her hands... you know, as if trying to figure out something from time ago.

– Was it black, the hair in her sample?

– Shit! How could I forget about it! – Arnie started nervously exploring a multitude of his pockets.

At the fourth try he extracted a small transparent plastic bag from the depths of his pants. Two copper-red hairs were shimmering in the light from the open doors of the theatre.

Arnie literally opened his mouth.

– Why? Where could Gregory come across that man? And... this hair is not unlike Gregory's, those...

– Look, Arnie,– I interrupted – Gregory's fur is not like this at all.

– He does have golden hairs here and there on his paws, slightly iridescent, not quite like fur.

– So much the reason for finding the man – Richard said.

– Do you think that he is the one who "made Gregory"? – Arnie asked.

– We must find out – Richard answered.

– Where does Eva live? Do you have her number, Arnie?

– I need to put the battery back, OK, here it is.

Richard looked, at the number, and Arnie removed the battery.

I chuckled to myself – The two really enjoy playing secret agents.

Richard, meanwhile, went to the box office window and exchanged a few green bills for a telephone receiver. He came back to us in a half a minute.

– By the way she sounds, something happened today and she is worried for the man. We shall meet her in fifteen minutes.

– Do you think the man is in a real danger? – I asked.

– He can react violently if Gibson's people try their "robot test" on him. Some of them, albeit illegally, carry guns.

– But still – I could not help asking – why all this cloak and dagger game?

Richard looked at me sternly.

– I've told you, *assume* I am paranoid.

I've got the message. We took a taxi to the Village.

Eva was waiting for us on the street.

– You know – she started by turning to Arnie and me – Jason, this man, thought you two were robots.

– Why on earth?

– Only a robot, he said, could have come up with such an idiotic question, you had asked, him, Arnie.

– Eva – Richard interfered,

–How did the RB robot get unfrozen at the robot quarters?

– A security man took Jason for a robot and attempted to check his ESR. Jason exploded, I guess you know how it was. The robot, apparently responding to the tenfold amplified string, unfroze and started reciting this Jenkins' nonsense about his creator.

– And what happened this afternoon?

– How do you know about today? From Helen, Jeremy?

– Simple, – I interfered – He reads your thoughts.

– Ah, these mischievous robot personality designers – Richard smiled – I'll talk to them later.

– Does psychology help working with robots?– I asked.

– Who said they are psychologists?– Richard responded, I would not trust a psychologist with this job, they come from the show business.

I raised my eyebrows questionably, but Eva interrupted me.

– Why, Richard? Why this string of yours? Why in public places?

– Was it Jeremy who suggested asking me?

– Yes, but how...

– I know him and he knows the answer.

6.7 Whose is Your Self?

Sigmund Freud discovered three structure components of "self".

1. *Id*: "... the dark, inaccessible part of our personality, what little we know of it we have learned from our study of the dream-work and of the construction of neurotic symptoms".

2. Ego: "attempts to mediate between id and reality".

3. Super-ego: "the installation of the super-ego can be described as a successful instance of identification with the parental agency."

The coherent performance of these relies upon the infrastructure of several *complexes*, especially upon *Oedipus complex.* Freud writes:

"A man should not strive to eliminate his complexes but to get into accord with them: they are legitimately what directs his conduct in the world".

Until recently, it went unnoticed how much the success of the original Freud's methodology was owing to his devoted assistant – Anna Emilia Karlsson, who stayed in shadow and had made every effort to remain invisible.

Her role was to prepare patients for psychoanalytic sessions performed by Freud. That was necessitated by disability of most patients to assimilate Freud's ideas due to the cognitive limitations of their minds. Anna Emilia had been conducting tutorials with the patients helping them to imprint proper complexes and correctly install the three egos into their psyche.

Freud's disciples greatly enriched the theory by discovering new complexes: the *Electra, Medea* and *Io* complexes, the *Sisyphus* and *Tantalus* complexes, the *Danaë, Perseus, Gorgon* and *Andromeda* complexes, the *Angantyr, Yrsa* and *Hljod* complexes, and, about ten years ago, the *Flatulus* complex that is complementary to the Oedipus complex.

However, being oblivious to the imprinting techniques by Karlsson, Freud's followers have been working with *unprepared* subjects who, understandably,

could not respond to psychoanalysis in accordance with the general guidelines of Freudian doctrines. Eventually, this has evoked an unjustified critique of the principles of Freudism.

The accidental discovery of four presumingly lost "complex imprinting" notebooks of Karlsson on the roof of an old building in Stockholm, buried in a tiny cache and amazingly well preserved, had marked a turning point in the development of the robot personification software.

Eva started her story.

– What do you think make our robots so popular, why do so many try to look like them? – Jeremy opened the conversation when three of us came to the Ajax Cafe next to the RB building.

– If somebody tells you: "Most people respond to whatever said to them by "Yes, and me..."... "

He turned to the couple at the next table.

– How would you reply?

Before they could react Jeremy answered for them.

– "Not me, I always... "

The two reddened with embarrassment.

– Nothing wrong with this – Helen smiled at the young people. We all think and talk mainly about ourselves, this is how we ensure our survival and project our personalities onto other people. But our robots are programmed, when speaking to you, to talk about *you*. We, at RB, did not invent this. It has always been the practice of priests, astrologists, fortune tellers, and, in more recent times, of psychotherapists.

– Psychotherapists – Jeremy interfered – turned this art into a science. You, probably, heard, Eva, of the old ELIZA program successfully imitating a psychotherapist. No knowledge was built into it what-so-ever, just a few phrase manipulation rules.

– Doesn't it tell you how much worth psychoanalysis is?

– Depends on how you look at it – Helen responded – Psychotherapists occasionally help people but psychoanalysis is no more a part of what *you* would call psychology, than astrology is a part of astronomy. Freud and Karlsson might have heard of Copernicus but, certainly, they had hardly been aware of of writings by Spalding.

– Who is Spalding?– I asked

– Douglas Spalding, I've learned this from Arnie, is the founder of the scientific psychology, He discovered imprinting in animals.

– But this was, probably, after Freud's death.

– Freud was about twenty when Spalding died, 1880 or something.

– So long ago! Anyway, Freudists are well insulated from any whiff of science, be it imprinting, quantum mechanics or molecular biology.

– Not anymore – Jeremy objected. – Freud style theories *are* encroaching into science these days. Using Bohr complementary principle, a group of psychomonists at Stanford have traced Helmholtz' definition of the free energy to the Flatulus complex of his grandmother on the maternal side that was recorded by the family physician when she was two yeas old.

– What? – I opened my mouth – What kind of science is this?

– A *new* kind of science. Yet, it has lead to an old-fashionably *falsifiable* prediction that a predisposition to Flatulus is encoded in the mitochondrial DNA.

– But what any Freudian complex may conceivably have to do with the free energy, be it Helmholtz or Gibbs.

– Helmholtz defined his free energy under constant volume, you know this better than I do, Eva. Flatulus is the Discworld god of wind. The Flatulus complex concerns maintaining constant volume of your...

– Stop it Jeremy – Helen interrupted him. Let Eva and me finish our ice cream.

What ever it is – I asked – Do your robots have the Flatulus complex installed into them?

– No – Jeremy smiled. It is even harder than installing the Oedipus complex with the present day robot architecture. But who cares? Robots perform in front of the *human* audience, they must leave their imprints on *human* psyche.

– Can your robots pass the Turing test?

– Depends on who is running the test. Put lots of "I" and "me" in the vocabulary and no psychotherapist will tell the difference. But what *we* do is sculpturing indigenous *robots'* personalities rather than imitating humans. There are no ready models here except for science fiction characters.

– Why doing it anyway?

– When the use of robots becomes widespread, people, maybe not all people, will get bored with only "yes sir" robots around them.

– I do not think – Helen said with a sly smile – that impersonating a robot is difficult. Look – and she made a theatrical gesture toward the window.

All heads turned there. Three big read haired men with unrealistic beards were engrossed into an animated conversation on the street, occasionally throwing glances at our table.

– These three are playing run away robots.– She said addressing everybody around.

When the men entered and approached a table next to us, Helen stood up and turned to them with an unnatural, exaggerated smile across her face.

– Welcome brothers – She said in a loud metallic voice.

–That's her, big woman! – exclaimed one of the men.

Another red haired man turned facing Helen, raised his hands up in the air and chanted loudly.

– Creators of devils – Hamilton and Jenkins

– Raised from hell, yes, raised from hell

– Satan's suns and Satan's daughters.

– Foul blood spill – Hamilton's beasts kill!

– Foul blood spill – Jenkins' beasts kill!

– Their beasts kill – foul blood spill!

Helen froze in surprise with the smile on her face.

– I've told you! – shouted the first man, jumped toward Helen and tried to tear her shirt.

– Satanic Rape! – Helen's shrill voice pierced the ears as a police siren.

Jeremy made a hesitant move of rising from the table. The third man put a restraining hand on his shoulder.

– Don't kill me! Don't spill my blood! Take the money! – Jeremy whimpered – his face a mask of fear, his trembling hand holding the wallet stretched toward man's bearded face. The man involuntary jerked back. The wallet fell down while half of the man's fake beard remained in Jeremy's clenched fist. Simultaneously, we heard something heavy fall on the floor and saw a gun at the man's feet.

– Murder! My heart! – Jeremy exclaimed standing up, picturesquely clasped his hands to his chest and fell down on his wallet next to the gun.

– Jesus Christ! Get the fuck out of here! – The man shouted, picked up his gun from the floor and dashed to the door.

He tripped over Helen's foot and went down with a crash dropping his gun. In a rush, the other two men collided with Helen.

– Blasphemous satanists! – She screamed on the top of her voice and landed with a grace of a professional wrestler on the man on the floor kicking away the gun and knocking off a bowl of ice cream from the table in the process.

The ice cream settled on what remained of the man's beard with two hundred pounds of "rape!" yelling Helen on the rest of him.

Two other men huddled over her but one after another they lost their footings and added to the commotion on the floor.

After a brief scramble with her legs wiggling in the air, kicking over a table and a couple chairs, Helen was back up on her feet, her shirt torn in half, a red hair wig in one hand and a black beard in the other.

She angrily threw down the wig and the beard on the floor, hitched up her pants and brushed her disheveled sandy hair off her face with her hands.

– They wanted to befoul my body! They thirsted for the blood of my children! I will not let them, I, Helen Hamilton! – She declared.

When the trio tried to raise themselves up from the floor, four guns were on them.

– Police! Freeze!

– Robot Control Agency, officers – The half-bearded man, apparently the leader, growled panting on hands and knees.

– Undercover mission. Top secret of national importance – The red wig fell down from under his chin.

I've finally recognized him – the man who triggered the loop at JFK.

– They killed that man!– somebody cried out – There's the gun! Call the ambulance!

All three were taken away in handcuffs. Helen went with Jeremy to the hospital in the ambulance car. She called me, everything was under control, she said.

Listening to Eva, Richard chuckled from time to time.

– I couldn't imagine. ... Bravo, Jeremy! Magnificent, Helen! ... Poor taxpayers.

– What the hell was magnificent, Richard? – Eva asked with rancor in her voice – Fighting RCA hoodlums is not *our* business.

– Fighting? *Who* was fighting? Even granting Gibson's opening move at Ajax was fortuitous, Helen and Jeremy played by the book. Imprinting the minds of the audience is *their* business.

– Jeremy? Does fainting count for much?

– Fainting? Guns do not jump out of the pockets by themselves. Of a different school, Jeremy is as adroit in his ways as Helen is in hers.

– What RCA will pay for? – Arnie asked Richard.

– To avoid a trial.

– But "Satan Controls Robot Controllers", "Undercover Satanic Rape Mission Foiled by the Intended Victim", etc. will be all over the newspapers tomorrow anyway.

– There are a few stronger cards than satanism. Kerr, I guess, will accuse the Robot Control Agency of instructing their operatives, disguised as robots, to orchestrate disorderly and criminal acts in New York, California and Massachusetts.

– The motive being...

– Legitimation of the RCA authority in these states.

– Will they sacrifice Gibson?

– They should, but I do not understand whose hand moves him on the board.

– Where does this satanic mumbo jumbo come from? – Eva asked.

– Two third of RCA operatives are former "robot hunters" from the Robot Prohibition League. For them robots are "Satan's children raised from hell" – Richard responded. – It is catchy.

A wry smile curved his lips.

– Those who cry Satan are, more often than not, his own disciples.

I turned to Eva with a slight bow. – With a due respect to the spies and hunters of His Infernal Majesty, you did not forget to take out the battery from you phone, I presume.

Richard answered for her.

– She's left her cell phone at home. *Do not* forget, I am paranoid.

– How could Helen manage three men? – I asked Richard. – Is she a judo black belt?

– Second dan, and not only this.

– And Jeremy?

– Jeremy? – Richard smiled – No, his is a *quite* different school.

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